On the Estimation of Nonneutral Production Technologies without Homothetic Separability*

SHUNAN ZHAO[1] EMIR MALIKOV[2] SUBAL C. KUMBHAKAR[3]

[1]Oakland University
[2]University of Nevada, Las Vegas
[3]Binghamton University

June 2, 2026

Abstract

This paper develops a novel approach for identifying production functions with factor-biased technical change without requiring homothetic separability. We show that for flexible parametric specifications, such as the translog, this restriction is unnecessary for identification. Leveraging information in the optimal variable input shares (instead of input ratios only), we provide a three-stage estimator within the canonical "proxy variable" tradition. Our framework permits a flexible input substitution structure, avoiding restrictions on production technology and associated firm behavior implied by homothetic separability. We find little empirical corroboration for imposing homothetic separability in three widely used manufacturing production datasets.

Keywords: Biased Technological Change, Factor-Augmenting Technology, Homothetic Separability, Production Function, Productivity, Proxy Variable, Translog

*Contacts: shunanzhao@oakland.edu (Zhao), emir.malikov@unlv.edu (Malikov), kkar@binghamton.edu (Kumbhakar).
We are grateful to Jing Zhang, Mark Tremblay, and Scott Orr for helpful discussions. We also thank participants at the 2025 NAPW and CES Meetings for feedback.

1 Introduction

To borrow from Krugman (1994), "productivity isn't everything, but, in the long run, it is almost everything," so it is unsurprising that interest in understanding productivity growth, its technological effects and sources has never waned among economists. Although straightforward in concept, measuring productivity is not trivial for myriad reasons, among which one is that, at a core, it is an unobservable residual. Identification is further complicated because firm productivity is correlated with input usage, inducing endogeneity in the estimation of production functions from which it is parsed out. Recent empirical evidence (see Raval, 2019; Dewitte et al., 2020; Oberfield and Raval, 2021) has also been increasingly pointing out that technological change is "biased," particularly toward labor, which is at odds with canonical structural production function estimators that assume Hicks-neutral technology with scalar productivity that affects all factors equally.

On this account, some headway has been made at extending structural methods to identify technologies with multi-dimensional factor-nonneutral productivity, with new models typically admitting labor-augmenting productivity besides the usual Hicks-neutral component (see Doraszelski and Jaumandreu, 2018; Zhang, 2019; Raval, 2019, 2023a; Demirer, 2025). These structural methodologies, however, rely on a framework where the production function is assumed—explicitly, or implicitly—to be homothetically separable in the fixed-vs-variable-factors partition of the firm's input vector. This restriction arises in different ways depending on the modeling approach: it may be an intrinsic property of the chosen functional form, as in the constant elasticity of substitution (CES),[1] explicitly imposed via parameter constraints, as in the restricted translog (RTL), or maintained as a shape restriction on an otherwise unrestricted nonparametric function. Homothetic separability is typically justified on the grounds that it provides sufficient structure to isolate labor-augmenting productivity from the variation in the variable input mix, which under the said restriction is orthogonal to factor-neutral technical change. Indeed, absent a parametric form specification, Demirer (2025) has shown that when imposing additional assumptions, such as (effective) variable inputs needing to be strictly either substitutes or complements as well as there being no unobserved demand heterogeneity, strong homothetic separability can facilitate nonparametric identification of all output elasticities. What remains understudied, however, are the full scope of theoretical implications of the homothetic separability assumption for production technology and the associated firm behavior, whether there is much empirical imperative for it, and if it is possible to dispense with this assumption altogether and still achieve full identification of nonneutral technology. This paper seeks to examine these questions.

Our main contribution is a new framework for identifying nonneutral production technologies without imposing homothetic separability, traditionally assumed in structural productivity analyses. We show that, for parametric specifications with nonneutral productivity that can affect labor and other inputs differentially, homothetic separability is *not* unequivocally necessary for structural identification. This is achievable if, drawing on Doraszelski and Jaumandreu's (2013) and Malikov and

[1] That is, CES satisfies homothetic separability by construction rather than through additional parameter restrictions.

Zhao's (2023) modeling paradigms, one fully exploits the parameter restrictions on the firm's static first-order conditions (FOCs) implied by a given form of the production function. To this end, we consider a (full) translog production function, but our identification arguments are more general and extend to other not-homothetically-separable specifications such as the Generalized Cobb-Douglas and the Generalized Leontief. Under the translog form, we show that a closed-form proxy function for labor-augmenting productivity derived from the FOCs for variable inputs remains independent of Hicksian productivity but is no longer a unique function of the variable input mix, rendering infeasible the go-to approach of relying on the variation in the labor-to-materials ratio alone to control for the nonneutral unobservable. Deviating from the existing methodologies, ours circumvents this problem by also making use of the additional information contained in the optimal levels of labor and material shares which are also rid of factor-neutral productivity. Resultant is a three-stage easy-to-implement identification scheme within the canonical Olley and Pakes (1996) and Levinsohn and Petrin (2003)-style structural "proxy variable" tradition.

Unlike Demirer's (2025) nonparametric and (even more popular in practice) also-parametric homothetically separable CES or RTL production frameworks with biased technical change (see Doraszelski and Jaumandreu, 2018, 2019; Zhang, 2019; Raval, 2019, 2023a; Malikov et al., 2024), our not-homothetically-separable production function permits a more flexible input substitution structure. It (i) does not impose orthogonality of the ease of variable-input substitution (between labor and materials) to the predetermined level of fixed input (capital), while forcing the latter to have equal elasticities of substitution (ES) with each one of those variable inputs, (ii) accommodates heterogeneous substitution elasticities between all factors, and (iii) allows factors to switch from complementarity to substitutability over the input support. These properties—what may seem like purely technical elements of the production technology—exact non-trivial behavioral restrictions on the firms. For instance, restricting capital to be equally easily substitutable with labor *and* intermediates goes against not only the historical empirical evidence but also undermines the soberly debated possibility of industrial robots displacing labor in the future.[2] Homothetic separability also implies that a firm must have a linear expansion path in the short run (when optimizing over variable inputs), with the conditional demand for labor relative to materials being scale-invariant, meaning that large and small firms must employ variable inputs in exactly the same proportion at given relative factor prices and productivity. We show that, as a direct implication of this scale invariance, the degree of returns to scale (RTS) at which the firm operates does not react to contemporaneous effective variable factor price shocks, ruling out endogenous changes in scale economies as a margin of adjustment. Changes in wages or materials prices, as well as labor productivity shocks, must therefore affect all firms alike, irrespective of their size or existing capital intensity. While the relevance of all these indirectly imposed theoretical restrictions on firm behavior likely depends on the specific context and industry, it is essential that researchers remain aware of their presence and implications.

Employing three widely used manufacturing production datasets from China, Chile, and Colom-

[2]E.g., see Herrera's (2025, June 30) recent Wall Street Journal Article about fast-pace automation of Amazon facilities.

bia, we find that data are generally inconsistent with homothetic separability and its implications. Directly, formal tests reject the RTL parameter restrictions implied by homothetic separability in most country-industry pairs. Indirectly, through examining the technological and behavioral restrictions on input substitution structure and scale elasticity, substitution elasticities between capital and each variable input generally differ, the degree of substitutability of labor and materials varies with the level of capital, and the RTS are not orthogonal to effective factor prices, all of which are inconsistent with the homothetic separability assumption. In addition, albeit not implied by homothetic separability *per se*, variable inputs can be both gross substitutes and complements.

The remainder of this paper is organized as follows. Section 2 discusses theoretical restrictions on technology and the associated firm behavior that the homothetic separability implies. Section 3 introduces a structural translog-based model of nonneutral production technology that is not homothetically separable and describes our identification strategy. Section 4 provides an estimation procedure, with the finite-sample performance of our proposed methodology in simulations reported in Section 5. Section 6 summarizes empirical results, testing the validity of homothetic separability in three widely used manufacturing data sets and reporting evidence in support of our production-function formulation. Section 7 concludes.

2 Homothetically Separable Production Functions

Let $\mathscr{I}$ be the firm's information set, based upon which it makes its current optimal production decisions each period. Now consider the following (stochastic) gross production function with non-factor-neutral technical change for this firm:

$$Y = F\left(K, e^{\varphi} L, M\right) e^{\omega+\eta}, \tag{2.1}$$

where $F(\cdot): \Re^3_+ \to \Re_+$ satisfies all standard properties of production functions (see Chambers, 1997, p.9), Y is the firm's output quantity, and inputs K, L and M are capital, labor, and materials, respectively. In the Gandhi et al. (2020) nomenclature, let physical capital K be a *non-flexible* input subject to adjustment frictions, with its optimal value predetermined by the firm prior to the current period. The remaining inputs $(L,M)'$ are *flexible*, which the firm chooses each period to maximize expected profits conditional on the already optimized K and other state variables. Thus, the firm chooses K solving its dynamic optimization problem in advance, whereas L and M are optimized contemporaneously given the level of K. Using the familiar textbook terminology, K is therefore "fixed" and $(L,M)'$ are "variable" when the firm makes production decisions each period and—in the same vein—the firm's optimization over variable inputs $(L,M)'$ conditional on fixed K is often styled as "short-run" (e.g., De Loecker et al., 2016).

Equation (2.1) features two-dimensional firm (log)-productivity: $\varphi \in \Re$ is labor-augmenting, and $\omega \in \Re$ is Hicks-neutral. Both are latent but known to the firm, $(\varphi, \omega)' \in \mathscr{I}$. Shock $\eta \in \Re$ is a pure ex-post noise, such as a measurement error in output, that is orthogonal to $\mathscr{I}$. In line with most literature (e.g., Ackerberg et al., 2015; Gandhi et al., 2020; Demirer, 2025), assume that markets for these variable factors are perfectly competitive, with homogeneous—but potentially time-varying—prices P^L and P^M common to all firms.[3] For convenience, also define "effective labor" as $\widetilde{L} \equiv e^{\varphi}L$ and, correspondingly, productivity-adjusted price (or "effective price") of labor as $\widetilde{P}^L \equiv e^{-\varphi}P^L$.

Recent methodological advances in the structural analysis of labor-augmenting technical change, including the seminal work by Doraszelski and Jaumandreu (2018) as well as subsequent papers by Zhang (2019), Raval (2019), and more recently Malikov et al. (2024) and Demirer (2025), rely on homothetic separability of the production function to achieve full identification thereof, along with the two productivity components. Specifically, the identifying restriction they all require is that $F(\cdot)$ is (strongly) "homothetic separable:"

$$F\left(K, e^{\varphi}L, M\right) = F\big(K, H(e^{\varphi}L, M)\big), \tag{2.2}$$

where $H(\cdot): \Re^2_+ \to \Re_+$ is twice-continuously differentiable and homogeneous of an arbitrary degree α_H. This functional separability is either an intrinsic feature or an explicitly imposed restriction, arising routinely regardless of whether, ultimately, the production function is modeled parametrically or nonparametrically, albeit its justification is usually made in the context of a generic $F(\cdot)$. Fundamentally, absent a parametric form specification, this restriction aids identification by ensuring that the variable input mix L/M contains information about φ that is independent of ω, and by providing sufficient structure on $F(\cdot)$ so that, together with additional assumptions such as a strict substitutes-or-complements restriction (Demirer, 2025), all output elasticities can be point-identified.

On the other hand, homothetic separability imposes non-trivial restrictions on production technology. It is easy to verify that functional separability in the partition $(K, (e^{\varphi}L, M))$ requires that the marginal rate of substitution (MRS) between two variable inputs is orthogonal to the level of fixed input, restricting the slope of isoquant curves in (L, M) space to being unaffected by the level of K:

$$\frac{\partial}{\partial K}\left(\frac{\frac{\partial}{\partial L}F(\cdot)}{\frac{\partial}{\partial M}F(\cdot)}\right) = 0 \qquad \Leftrightarrow \qquad \frac{\partial \ln\left(\frac{\partial}{\partial L}F(\cdot)\right)}{\partial \ln K} = \frac{\partial \ln\left(\frac{\partial}{\partial M}F(\cdot)\right)}{\partial \ln K}, \tag{2.3}$$

or equivalently, requiring that elasticities of the marginal products of L and M with respect to K be equal. The above is, essentially, how input "separability" is defined in production theory. However, assuming that this separability is *homothetic* imposes further restrictions on input substitution structure by going beyond just regulating the slope of isoquants at any given point. It also constrains their curvature, further limiting how inputs are mutually substitutable within production. Namely, the restriction in (2.2) forces K to have equal ES with both variable inputs, whereby the isoquant surface is symmetric in the two radial directions from K towards L and M. This can be problematic. For example, while industrial automation in manufacturing processes replaces labor—unskilled workers, in particular—it hardly enables producers to substitute away from intermediates (e.g., wood and plastic) with the same ease as the latter are essential "ingredients" of output (e.g., tables), the

[3] Constancy of prices across firms is unimportant so long as cross-firm heterogeneity therein is exogenous to firms.

material composition of which is largely independent of the production methods. The assumption of equally "easy" substitution possibility between capital and labor and materials is also at odds with early empirical studies (e.g., Humphrey and Moroney, 1975; Berndt and Wood, 1975; Moroney and Toevs, 1977; Gamponia and Brown Jr, 1982; Chung, 1987).[4] We formalize this, and other, implications of homothetically separable production functions below.

Consider the two common ES measures for production with more than two inputs: the Allen-Uzawa partial elasticity of substitution and the Morishima elasticity of substitution (for a textbook treatment, see Chambers, 1997). Among these two, the Allen-Uzawa measure is perhaps the most widely used in empirical work. It is symmetric, meaning that the ES between any two inputs, such as K and L, is equal to that between L and K, and it reduces to the familiar formula when there are only two factors. Given a production function $F(X_1,\ldots,X_J)$, for any X_p and X_q it is defined as

$$\mathrm{AES}_{pq} = \frac{\sum_s X_s F_s}{X_p X_q} \frac{\mathrm{C}_{pq}}{|\overline{\mathrm{B}}|}, \tag{2.4}$$

where $F_s = \frac{\partial}{\partial X_s} F(\cdot)$ is the marginal product of X_s, $\overline{\mathrm{B}}$ is the bordered Hessian matrix, and C_{pq} is the cofactor of $\overline{\mathrm{B}}$ corresponding to X_p and X_q. Its popularity notwithstanding, the AES has been criticized for being uninformative of the ease of input substitution (see Blackorby and Russell, 1981, 1989). To address this concern, we also consider the Morishima ES, which preserves essential characteristics of the original Hicksian concept. Unlike the Allen-Uzawa, the Morishima-Blackorby-Russell elasticity is asymmetric and provides directional information about input substitution. Namely, under duality it measures how the factor demand X_p/X_q changes in response to the relative price change originating from the price of X_p. This response may differ when the relative price change originates from the price of X_q as the input ratio may be more sensitive to changes in one factor price than the other. In the primal formulation, this ES is defined as

$$\mathrm{MES}_{pq} = \frac{F_p}{X_q} \frac{\mathrm{C}_{pq}}{|\overline{\mathrm{B}}|} - \frac{F_p}{X_p} \frac{\mathrm{C}_{pp}}{|\overline{\mathrm{B}}|}. \tag{2.5}$$

With both ES measures, a negative value indicates that the inputs are net complements, whereas a positive value means they are net substitutes.[5] Also note that, since ES is scale-invariant, any one input's ES with L is the same as that with $\widetilde{L}$ (see Online Appendix O1).

Proposition 1 *Let a twice-continuously differentiable production function $F(\cdot)$ be homothetically separable in $(e^{\varphi}L, M)$ from K per the restriction* (2.2). *Then, Allen-Uzawa and Morishima substitution elasticities between K and either within-subfunction input L or M are equal:* $\mathrm{AES}_{KL} = \mathrm{AES}_{KM}$

[4] Of note, separability between K and $(L,M)'$ also refutes the premise of vast empirical literature estimating value-added production functions, which typically relies on the Leontief-form assumption that it is the capital and labor that are separable from intermediates (see Ackerberg et al., 2015).

[5] From (2.4)–(2.5), $\mathrm{MES}_{pq} = \frac{X_p F_p}{\sum_s X_s F_s}\left(\mathrm{AES}_{pq} - \mathrm{AES}_{pp}\right)$. Since the own-price elasticity AES_{pp} is negative by definition, it is apparent that two inputs can be Allen-Uzawa complements but Morishima substitutes. But if the two are Allen-Uzawa substitutes, then they are also Morishima substitutes. For the details, see above-cited references.

and $\text{MES}_{KL} = \text{MES}_{KM}$.[6]

Proof. See Appendix A. □

Of note, the above technological implication of homothetic separability for the firm's input substitution structure does *not* depend on the degree of flexibility of factors, such as that K is fixed and $(L,M)'$ are variable. In practice, however, the restriction in (2.2) is imposed jointly with the "timing" assumption that inputs within the $H(\cdot)$ subfunction (labor and materials) are variable and those outside it (capital) are fixed in the short run. Together, these translate into additional, behavioral restrictions on the firm's production decisions.

Namely, we have $F(K,Q)$ where Q can be viewed as an intermediate product with its own homothetic subproduction $Q = H(e^{\varphi}L, M) = M^{\alpha_H} H(e^{\varphi}L/M, 1)$ which is combined with K to yield the firm's ultimate output. Homogeneity of the $H(\cdot)$ subfunction implies that changing the level of $(L,M)'$ is equivalent to changing the variable input mix L/M and its scale via M^{α_H}. As such, when optimizing the firm can first choose input *composition* for the subproduction (via the L/M ratio) and then optimize over the *scale* of the intermediate Q and K. In other words, the restriction in (2.2) allows the separation of mix and scale decisions for variable inputs, ultimately rendering the short-run input mix choices invariant to the scale of production. So long as the level of K is fixed, as formally shown below, homothetic separability can also impose orthogonality of the ES between variable inputs L and M to the scale of K, implying that firms utilizing both a lot and a little capital—a predetermined decision—have the same composition of variable inputs in their production when facing the same effective input prices.

These behavioral restrictions become more apparent if we consider an equivalent representation of the firm's homothetically separable technology. Specifically, with $\Lambda \equiv \mathbb{E}[e^{\eta}|\mathscr{I}] = \mathbb{E}[e^{\eta}]$, let $Y^* = F(\cdot)e^{\omega}\Lambda$ be the firm's expected, or "planned," output level for the current period. Then, since $F(\cdot)$ is strictly monotone, we can rewrite it in the expectation (conditional on $\mathscr{I}$) under homothetic separability in (2.2) as

$$F^{-1}\big(\underbrace{e^{-\omega}Y^*/\Lambda}_{\widetilde{Y}^*}, K\big) = H\big(e^{\varphi}L, M\big). \tag{2.6}$$

Basically, the subfunction $H(\cdot)$ is the textbook "short-run production function" for the firm's net[7] planned output $\widetilde{Y}^*$ producible in the current period given the fixed level of K, subject to which the firm makes its optimal variable input allocations. Since the left-hand-side intermediate product $Q \equiv F^{-1}\big(\widetilde{Y}^*, K\big)$ is a one-to-one mapping $\widetilde{Y}^* \to Q \mid K$ for a given capital level, the above representation of the firm's production frontier in the short run provides apparent connection to a familiar case of homothetic production functions.

[6] The Morishima elasticity of substitution is directional, with the first index denoting the input whose price changes. Hence, MES_{KL} and MES_{KM} respectively measure how the capital–labor and capital–materials ratios respond to changes in the price of capital, which is the relevant comparison under homothetic separability. In contrast, MES_{LK} and MES_{MK} focus on changes in the prices of labor and materials. Because these elasticities are driven by different sources of price variation, they need not coincide even under homothetic separability. See the proof for details of derivation.

[7] Net of factor-neutral productivity ω and a scaling factor Λ, that is.

Homotheticity of $H(\cdot)$ imposes that the firm's variable input choices always change equiproportionately with changes in its intermediate production scale Q. Thus, the firm's choice of the $\left(\widetilde{L},M\right)$ mix (the L/M ratio) depends only on their relative effective factor prices and not on Q (or equivalently, not on the level of K or $\widetilde{Y}^*$). In other words, along any expansion of output in the short run, the firm expands $\widetilde{L}$ and M proportionally to one another. The associated expansion path is therefore linear going through the origin in $\left(\widetilde{L},M\right)$ space, and the demand for labor relative to materials is scale-invariant. This follows immediately from recognizing that the ex-ante short-run variable cost (VC) function of a risk-neutral firm with homothetically separable production function is also functionally separable, taking the form

$$VC\left(\widetilde{Y}^*,K,\widetilde{P}^L,P^M\right) = \left[F^{-1}\left(\widetilde{Y}^*,K\right)\right]^{1/\alpha^H} VC^1\left(\widetilde{P}^L,P^M\right), \tag{2.7}$$

where VC^1 is the variable cost of 1 unit of the intermediate product Q (see Appendix B.1 for derivation). Then, per Shephard's lemma, the firm's optimal relative demand for effective labor expectedly only depends on (productivity-adjusted) inputs prices:

$$\frac{\widetilde{L}}{M} = \frac{\frac{\partial}{\partial \widetilde{P}^L} VC^1\left(\widetilde{P}^L,P^M\right)}{\frac{\partial}{\partial P^M} VC^1\left(\widetilde{P}^L,P^M\right)} \quad \Leftrightarrow \quad \frac{L}{M} = \frac{\frac{\partial}{\partial P^L} VC^1\left(e^{-\varphi}P^L,P^M\right)}{\frac{\partial}{\partial P^M} VC^1\left(e^{-\varphi}P^L,P^M\right)}, \tag{2.8}$$

with any scale dependence in the short-run relative demand for variable factors being assumed away:[8] large and small firms must employ L relative to M in exactly the same proportion at a given relative factor price and productivity φ. Evidently, the ease of substitutability between L and M in the short run will also be unaffected by the scale of fixed K.

Further, because the firm's optimal (L,M) mix does not adjust with scale, as a direct implication, the local degree of scale economies at which the firm operates does not react to changes in the effective variable input prices.[9] As such, differences in the effective labor price do not induce systematic differences in RTS across firms. This invariance of scale economies to productivity-adjusted wages $\widetilde{P}^L$ is of particular interest because the variation in the latter can originate from labor-augmenting technical change. Thus, the structure in (2.2) rules out endogenous changes in RTS as a margin of adjustment to effective factor price shocks. We summarize these implications in the following proposition.

Proposition 2 *Assume that, given its productivity profile* (ω,φ)*, a firm with a twice-continuously differentiable production function* $F(\cdot)$ *satisfying the homothetic separability restriction* (2.2) *minimizes short-run variable costs conditional on fixed* K*, taking the expected output level* Y^* *and input prices* (P^L,P^M) *as given. Then,*

(a) Allen-Uzawa and Morishima substitution elasticities between variable inputs $(L,M)'$ *are invari-*

[8]This scale independence of homothetic production functions is well-known, and in the context of a CES specification, the work aimed at relaxing it dates at least as far back as Sato (1974, 1975, 1977) who pioneered "nonhomothetic CES" functions.

[9]See Appendix B.2 for additional discussion.

ant to the level of fixed input K;

(b) *the degree of RTS is invariant to the effective labor price $e^{-\varphi}P^L$.*

Proof. The proof is straightforward using dual definitions of ES and RTS and follows immediately from (2.7). Specifically, substitution elasticities between L and M are measurable using costs via (see Blackorby and Russell, 1981)

$$\mathrm{AES}_{LM} = \frac{VC \times VC_{LM}}{VC_L \times VC_M},\ \mathrm{MES}_{LM} = \frac{P^L VC_{LM}}{VC_M} - \frac{P^L VC_{LL}}{VC_L},\ \mathrm{MES}_{ML} = \frac{P^M VC_{LM}}{VC_L} - \frac{P^M VC_{MM}}{VC_M}, \quad (2.9)$$

where $VC_X \equiv \frac{\partial VC}{\partial P^X}$ and $VC_{XZ} \equiv \frac{\partial^2 VC}{\partial P^X \partial P^Z}$ for $(X,Z)' \in \{L,M\}$. Because under homothetic separability in the fixed-vs-variable-factors partition, the variable cost function is multiplicatively separable in the short-run scale term $\left[F^{-1}\left(\widetilde{Y}^*,K\right)\right]^{1/\alpha^H}$ per (2.7), the above ES expressions simplify to

$$\mathrm{AES}_{LM} = \frac{VC^1 \times VC^1_{LM}}{VC^1_L \times VC^1_M},\ \mathrm{MES}_{LM} = \frac{P^L VC^1_{LM}}{VC^1_M} - \frac{P^L VC^1_{LL}}{VC^1_L},\ \mathrm{MES}_{ML} = \frac{P^M VC^1_{LM}}{VC^1_L} - \frac{P^M VC^1_{MM}}{VC^1_M}. \quad (2.10)$$

With no component in these ES measures depending on fixed K, it follows that

$$\frac{\partial \mathrm{AES}_{LM}}{\partial K} = \frac{\partial \mathrm{MES}_{LM}}{\partial K} = \frac{\partial \mathrm{MES}_{ML}}{\partial K} = 0, \quad (2.11)$$

which completes the proof of part (a).

To prove part (b), owing to the separable form of $VC(\cdot)$, also recognize that both the output and capital elasticity of variable cost—that is, $\epsilon^Y \equiv \frac{\partial \ln VC}{\partial \ln \widetilde{Y}^*} = \frac{\partial \ln VC}{\partial \ln Y^*}$ and $\epsilon^K \equiv \frac{\partial \ln VC}{\partial \ln K}$, respectively—depend only on $\left(K, \widetilde{Y}^*\right)'$. Thus, we have that $\frac{\partial \epsilon^Y}{\partial \widetilde{P}^L} = 0$ and $\frac{\partial \epsilon^K}{\partial \widetilde{P}^L} = 0$, entailing

$$\frac{\partial \mathrm{RTS}}{\partial \widetilde{P}^L} = \frac{\partial}{\partial \widetilde{P}^L}\left(\frac{1-\epsilon^K}{\epsilon^Y}\right) = 0, \quad (2.12)$$

where $\mathrm{RTS} = \frac{1-\epsilon^K}{\epsilon^Y}$ is a dual measure based on variable costs (see Caves et al., 1981). □

Of note, scale invariance of the variable input mix may have further ramifications for firm behavior, impacting factor-to-output price pass-through as well as shutdown considerations. Concretely, on the supply side, homothetic separability can rule out an RTS channel in the transmission of factor price or factor-specific productivity shocks onto the firm's output price. In addition, it can force such shocks to have no affect on the firm's shutdown condition in the short run. Since these are not the focus of our empirical investigation of homothetic separability, we relegate their discussion to Appendix B.3.

3 Translog Model with Nonneutral Productivity

Without specifying its form, a nonneutral production function $F(\cdot)$ can be nonparametrically identified under the homothetic separability restriction (2.2) and a few additional assumptions, such as effective variable inputs needing to be either strictly substitutes or complements and there being no unobserved demand heterogeneity (see Demirer, 2025, for details). In what follows, we provide a new framework to structurally point-identify $F(\cdot)$ without requiring that it be homothetically separable in inputs. To accomplish this, echoing Doraszelski and Jaumandreu's (2013) modeling approach, we fully embrace the assumed parametric specification of nonseparable $F(\cdot)$ in explicitly utilizing known functional forms of the firm's static FOCs and productivity proxy functions for ω and φ that they imply. Notably, the popular also-parametric production frameworks with biased technical change that have homothetically separable CES or RTL forms (Doraszelski and Jaumandreu, 2018, 2019; Malikov et al., 2024) equally rely on the assumed functional form of the production function for identification. But the matter of fact is, for parametric specifications, homothetic separability is *not* an unequivocal prerequisite for structurally identifying nonneutral production technologies, notwithstanding the default use of such functional forms.

We consider a (full) translog specification for the production function $F(\cdot)$ with nonneutral technical change in (2.1):

$$\ln F\left(K,\widetilde{L},M\right) = \beta_k k + \beta_l \widetilde{l} + \beta_m m + \beta_{kl} k\widetilde{l} + \beta_{km} km + \beta_{lm} \widetilde{l}m + \tfrac{1}{2}\beta_{kk}k^2 + \tfrac{1}{2}\beta_{ll}\widetilde{l}^2 + \tfrac{1}{2}\beta_{mm}m^2, \tag{3.1}$$

where, throughout the paper, lower-case Latin-alphabet letters denote logarithms of their upper-case counterparts.[10] This log-quadratic specification (i) accommodates non-constant, heterogeneous elasticities of substitution across inputs, (ii) does not force the MRS between variable labor and material inputs to be orthogonal to the fixed level of capital, and (iii) does not impose scale independence on the variable input ratio. We formalize this and other, already stated assumptions about firm behavior and the timing of its input choices below.

Assumption 1 *Nonneutral production technology relating the firm's output Y and inputs $(K,L,M)'$ is represented by the stochastic eq.* (2.1)*, with $F(\cdot)$ taking the translog form in* (3.1)*. The idiosyncratic ex-post shock $\eta \perp\!\!\!\perp \mathscr{I}$ is mean-zero. Among the factors, input K is fixed, with its optimal value determined dynamically by the firm in the previous period, whereby $K \in \mathscr{I}$. Each period, the firm maximizes its expected short-run profits conditional on K and other state variables in $\mathscr{I}$ by choosing variable inputs $(L,M)'$, the markets for which are perfectly competitive with exogenous prices P^L and P^M.*

Under imperfect competition in the output market, the firm's optimality conditions for the usual

[10] Since the constant is not separably identified from the unobserved ω, the intercept is normalized to 0.

short-run (expected) restricted profit maximization are

$$\mathbb{E}_t\left[\mathrm{MR}(P^Y)\right]\tfrac{\partial}{\partial L}F\left(K,e^{\varphi}L,M\right)e^{\omega}\Lambda=P^L \quad \text{and} \quad \mathbb{E}_t\left[\mathrm{MR}(P^Y)\right]\tfrac{\partial}{\partial M}F\left(K,e^{\varphi}L,M\right)e^{\omega}\Lambda=P^M, \tag{3.2}$$

where $\mathbb{E}_t\left[\mathrm{MR}(P^Y)\right]\equiv\mathbb{E}\left[P^Y\left(1+\frac{\partial\ln\mathscr{P}}{\partial\ln F}\right)\mid\mathscr{I}\right]$ is the expected marginal revenue (marked-up price), with $\mathscr{P}$ being the firm's perceived residual (inverse) demand curve. Under the translog assumption, the ratio of these FOCs expressed in logs, which dictates that the relative nominal demand for the firm's variable inputs must equal the ratio of their input elasticities, takes the following form:[11]

$$\frac{P^L L}{P^M M}=\frac{\frac{\partial}{\partial\ln L}\ln F\left(K,\widetilde{L},M\right)}{\frac{\partial}{\partial\ln M}\ln F\left(K,\widetilde{L},M\right)}=\frac{\beta_l+\beta_{kl}k+\beta_{lm}m+\beta_{ll}l+\beta_{ll}\varphi}{\beta_m+\beta_{km}k+\beta_{mm}m+\beta_{lm}l+\beta_{lm}\varphi}. \tag{3.3}$$

This equation helps isolate labor-augmenting φ from Hicks-neutral ω. Since $\frac{P^L L}{P^M M}$ depends on one unobservable φ only, the latter can be pinned down off additional information in the (nominal) input ratio besides just the variation in input levels. What is important, this fundamental result holds without homothetic separability and is true for any $F(\cdot)$. The extra mileage we get from the translog specification is the explicit functional form for the correspondence between φ and the observables, obtainable by solving (3.3) for φ:

$$\varphi=\frac{\beta_l+\beta_{kl}k+\beta_{lm}m+\beta_{ll}l-\left(\beta_m+\beta_{km}k+\beta_{mm}m+\beta_{lm}l\right)\frac{P^L L}{P^M M}}{\beta_{lm}\left[\frac{P^L L}{P^M M}\right]-\beta_{ll}}. \tag{3.4}$$

Because we do *not* impose homothetic separability on the firm's technology, the above proxy function for φ takes on a more complicated form than analogous expressions implied by the existing CES or RTL models that do. Concretely, imposing the restriction (2.2) onto the translog technology like Doraszelski and Jaumandreu (2019), Jaumandreu and Mullens (2024) and Malikov et al. (2024) by restricting parameters so that $\beta_{kl}=\beta_{km}=0$ and $\beta_{mm}=\beta_{ll}=-\beta_{lm}$,[12] the φ proxy in (3.4) simplifies under the RTL to

$$\varphi=\frac{\beta_l-\beta_m\left[\frac{P^L L}{P^M M}\right]}{\beta_{lm}\left(1+\frac{P^L L}{P^M M}\right)}-(l-m). \tag{3.5}$$

Alternatively, take the popular CES form as considered by Doraszelski and Jaumandreu (2018) and Zhang (2019), whereby $F\left(K,\widetilde{L},M\right)=\left[\alpha_k K^{\rho}+\widetilde{L}^{\rho}+\alpha_m M^{\rho}\right]^{\frac{\nu}{\rho}}$, in which case it can be shown that the corresponding φ proxy is

$$\varphi=\frac{1}{\rho}\ln\alpha_m+\frac{1}{\rho}\ln\left[\frac{P^L L}{P^M M}\right]-(l-m). \tag{3.6}$$

Contrasting with expressions in (3.5)–(3.6) for homothetically separable specifications, our φ

[11] From (3.3), it is evident that under the full translog specification the firm's relative variable input demand L/M is *not* forced to be scale-independent in the short run: it depends on a fixed level of capital k and indirectly—through *levels* of *all* three inputs $(k,l,m)'$—on the scale of production.

[12] The restriction $\beta_{mm}=\beta_{ll}=-\beta_{lm}$ implies that $H(\cdot)$ is homogeneous of (arbitrary) degree $\beta_l+\beta_m$.

proxy under the full translog in (3.4) is conditional on *all*—dynamic and static—input levels, as well as admits many more production-function parameters. Of note, it is also not a unique function of production-function parameters which it admits as it is homogeneous of degree zero in these coefficients: the mapping from φ to $\left(k, l, m, \frac{P^L L}{P^M M}\right)'$ obtained from the ratio of the FOCs is not one-to-one.[13] This means that the typical approach of combining such a φ proxy with its law of motion to form one of the identifying equations, as for example Doraszelski and Jaumandreu (2018) do using the equation (3.6) under the CES specification, is no longer feasible. Incidentally, the φ proxy under the RTL in (3.5) also suffers from the same problem, which Doraszelski and Jaumandreu (2019) and Jaumandreu and Mullens (2024) get around by directly transforming φ out of the production function via substitution using the proxy. In their case, aided—in no small measure—by the homothetic separability parameter restrictions, this produces a model of well-specified tractable form. Without homothetic separability assumption like in our case, the resultant would be a complex nonlinear form for which no clear simplification is apparent, obscuring whether every production-function parameter is identifiable from it. To point-identify all technology parameters as well as both productivity components in our nonseparable translog model we, therefore, pursue a different identification strategy. As we detail in Section 3.1, opportunely, the variation in the labor and material shares (in total revenue) contain additional information that we can use to pin down all parameters. To this end, we rely on both variable input share equations in levels as opposed to only their ratio to proxy for φ to achieve identification.

Of note, since the homothetically separable RTL form is a special case of the full translog, we can directly relate their corresponding φ inversions of the ratio of FOCs. Denoting the proxy under homothetic separability in (3.5) as φ^{HS}, the one under the full translog in (3.4) can be written as

$$\varphi = \underbrace{\frac{\beta_{lm}\left[1+\frac{P^L L}{P^M M}\right]}{\beta_{lm}\left[\frac{P^L L}{P^M M}\right]-\beta_{ll}}}_{a} \times \varphi^{HS} + \underbrace{\frac{(\beta_{lm}+\beta_{ll})\, l - (\beta_{lm}+\beta_{mm}) m \left[\frac{P^L L}{P^M M}\right]}{\beta_{lm}\left[\frac{P^L L}{P^M M}\right]-\beta_{ll}}}_{b} + \underbrace{\frac{\beta_{kl} k - \beta_{km} k \left[\frac{P^L L}{P^M M}\right]}{\beta_{lm}\left[\frac{P^L L}{P^M M}\right]-\beta_{ll}}}_{c(k)}.$$

This decomposition shows that imposing the homothetic separability assumption alters both the scale (a) and location ($b + c(k)$) of the labor-augmenting productivity function. In the RTL, separability from capital via $\beta_{kl} = \beta_{km} = 0$ forces $c(k) = 0 \ \forall k$, whereas homogeneity in labor and materials via $\beta_{ll} = \beta_{mm} = -\beta_{lm}$ forces $a = 1$ and $b = 0$. It is also apparent that the equivalence between φ and φ^{HS} can be tested by directly testing these parameter restrictions on the production function, which we do in the empirical application.

Next, we flesh out further structural assumptions that facilitate point identification of the translog production function in (3.1). Note, however, that the translog form is not the only available choice of a specification that does not satisfy homothetic separability in (2.2). In Supplementary Online Ap-

[13] To emphasize, this is not a simple issue resolvable using a normalizing restriction on parameters akin to assuming a zero intercept in the production function because the latter is not separable from the mean of ω. Every single parameter entering (3.4) is a slope coefficient in the underlying production function (3.1). As such, any normalization thereof would be tantamount to imposing an arbitrary untested restriction on input elasticities and cross-input substitutability.

pendix O2, we demonstrate the generality of our methodology for parametric nonneutral production functions by also considering two alternative forms for $F(\cdot)$: the Generalized Cobb-Douglas that is nonseparable in inputs and the Generalized Leontief that is neither homothetic nor separable.

Assumption 2 *Firms know contemporaneous levels of their productivity at the time of making production decisions: $(\omega,\varphi)' \in \mathscr{I}$. Writing $\mathbb{L}$ as the lag operator, these (log-)productivity components follow controlled first-order Markov processes, with the following linear projection representations:*

$$\varphi = \alpha_1 \mathbb{L}[\varphi] + \alpha_2 \mathbb{L}[Z_1] + \epsilon, \tag{3.7}$$

$$\omega = \gamma_0 + \gamma_1 \mathbb{L}[\omega] + \gamma_2 \mathbb{L}[Z_2] + \varepsilon, \tag{3.8}$$

where zero-mean innovations ϵ and ε are mean-orthogonal to the previous-period information set $\mathbb{L}[\mathscr{I}]$ inclusive of productivity shifters $\left(\mathbb{L}[Z_1], \mathbb{L}[Z_2]\right)'$ which may be identical.

Assumption 2 about unobservables is a standard assumption in the literature (e.g., Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020). For identification, we require a normalization that φ is zero-mean (hence, no intercept) because the constant of labor-augmenting productivity is not separably identifiable from the slope coefficient on $\widetilde{l} = \varphi + l$ in the production function.[14] On the other hand, the mean of ω (the intercept γ_0) is identified relative to a normalized zero intercept in the production function (3.1).

We allow firms to have monopolistic power in the output market, which has a close connection to recent developments in the literature and, particularly, studies focusing on markup estimation (see, e.g., Doraszelski and Jaumandreu, 2019; Bond et al., 2021; Blum et al., 2024; Raval, 2023a). Specifically, following De Loecker (2011), Grieco et al. (2016), and Zhao et al. (2020), we assume the Dixit-Stiglitz demand structure for Y. Our choice of such a straightforward demand formulation is motivated by its tractability. Most production datasets do not contain information on output quantity Y or prices P^Y but, instead, only report the firm's revenue income $R = P^Y Y$. The constant-elasticity demand assumption enables us to formalize the "revenue production function" that allows identification of all parameters of the "quantity production function" without needing to observe neither quantities nor prices of firm output.

Assumption 3 *Firms are monopolistically competitive in the output market and face the following (stochastic) inverse demand function:*

$$P^Y = \overline{P} \left(\frac{Y}{\overline{Y}} \right)^{1/\sigma} e^{\rho' \chi + \xi}, \tag{3.9}$$

where $\overline{P}$ is the current price index—the average price—for a given industry / segment, $\overline{Y}$ is the corresponding quantity index that plays the role of an aggregate demand shifter within an industry / segment,

[14] This is equivalent to a unit-value normalization on the labor distribution parameter in the CES case in Doraszelski and Jaumandreu (2018) or Zhang (2019).

χ captures observed demand shocks specific to a firm, $\xi \perp\!\!\!\perp \mathscr{I}$ is a zero-mean innovation unknown to firms ex ante, and $\sigma < 0$ is the price elasticity of demand.

By the definition of $\mathscr{I}$, we have that $\left(\overline{P}, \overline{Y}, \chi'\right)' \in \mathscr{I}$. Industry/sector-level price indexes $\overline{P}$ are often available in production datasets, and $\overline{Y}$ can be constructed, for example, as a market-share weighted average of all firms' outputs (De Loecker, 2011). Assuming that unobserved demand heterogeneity ξ is an independent noise is stronger than the identification needs, can be relaxed, and is done for simplicity. But we should note that the constant-price-elasticity demand in Assumption 3 does rule out unobserved *correlated* heterogeneity in the demand. If such heterogeneity exists and is correlated with input allocations (such as the choice of materials), admittedly, it may bias the estimated output elasticities; see Jaumandreu and Yin (2018). Production function estimators that control for correlated heterogeneity in prices (e.g., Guillard et al., 2018; Zhao et al., 2020; Blum et al., 2024) have done so focusing on simpler settings with only factor-neutral productivity. In our case, due to the complexities of accommodating multidimensional nonneutral heterogeneity in production, we do not seek to incorporate this feature to maintain tractability.

As the output quantity Y entering the production function is unobserved, we replace it with observable revenue R using the formulated demand structure. Whereas the canonical models tackle this issue by assuming perfect competition, whereby $P^Y = \overline{P}$ so that the output quantity can be recovered by deflating R using aggregate price indices,[15] in our framework the deflated revenue production function in logs is given by (per Assumption 3)

$$\widetilde{r} = \left(1 + \tfrac{1}{\sigma}\right) \ln F\left(K, \widetilde{L}, M\right) + \left(1 + \tfrac{1}{\sigma}\right)\omega - \tfrac{1}{\sigma}\overline{y} + \rho'\chi + \xi + \left(1 + \tfrac{1}{\sigma}\right)\eta, \tag{3.10}$$

where $\widetilde{r} \equiv \ln\left(\frac{P^Y Y}{\overline{P}}\right)$. Point identification of (3.10) above will deliver the production function $F(\cdot)$, both productivity components $(\varphi, \omega)'$ and the demand parameters $(\sigma, \rho')'$.

Under Assumptions 1 and 3, a firm's short-run optimization problem is

$$\max_{M,L} \quad \mathbb{E}[R|\mathscr{I}] - P^M M - P^L L = \frac{\overline{P}}{\left[\overline{Y}\right]^{1/\sigma}} \left[F\left(K, \widetilde{L}, M\right) e^{\omega}\right]^{1+\frac{1}{\sigma}} \Lambda^* e^{\rho'\chi} - P^M M - P^L L, \tag{3.11}$$

where $\Lambda^* \equiv \mathbb{E}\left[e^{\eta^*}|\mathscr{I}\right] = \mathbb{E}\left[e^{\eta^*}\right]$ is a mean of composite error $\eta^* \equiv \left(1 + \frac{1}{\sigma}\right)\eta + \xi$, and it appears because $(\eta, \xi)'$ are not observed *ex ante* when the firm makes its production choices. Writing input revenue share as $S^X \equiv \frac{P^X X}{R}$ for $X = \{L, M\}$, the analogues of short-run FOCs (3.2) under our demand structure written in a share form yield two stochastic equations:

$$S^L = \left(1 + \tfrac{1}{\sigma}\right) \frac{\partial \ln F\left(K, \widetilde{L}, M\right)}{\partial \ln L} \Lambda^* e^{-\eta^*} \quad \text{and} \quad S^M = \left(1 + \tfrac{1}{\sigma}\right) \frac{\partial \ln F\left(K, \widetilde{L}, M\right)}{\partial \ln M} \Lambda^* e^{-\eta^*}, \tag{3.12}$$

which—as is easy to confirm—when taking their ratio, yield the expression (3.3) providing a non-stochastic proxy for φ in (3.4).

[15]A special case of perfectly competitive output markets is considered in Supplementary Online Appendix O3.

3.1 Identification

We seek to identify parameters of the translog revenue production function in (3.10) subject to (3.1):

$$\widetilde{r} = \beta_k^* k + \beta_l^*[\varphi + l] + \beta_m^* m + \beta_{kl}^* k[\varphi + l] + \beta_{km}^* km + \beta_{lm}^*[\varphi + l]m + \tfrac{1}{2}\beta_{kk}^* k^2 + \tfrac{1}{2}\beta_{ll}^*[\varphi + l]^2 + \tfrac{1}{2}\beta_{mm}^* m^2 + \left(1 + \tfrac{1}{\sigma}\right)\omega - \tfrac{1}{\sigma}\overline{y} + \rho'\chi + \eta^*, \tag{3.13}$$

where $\beta^* \equiv \left(1 + \frac{1}{\sigma}\right)\beta$ for any "β" translog coefficient and, recall, $\eta^* \equiv \left(1 + \frac{1}{\sigma}\right)\eta + \xi$ is a composite noise.

Our three-stage identification strategy combines structural methods with dynamic panel techniques. We draw on the "proxy variable" framework in that we use a firm's static FOCs to express the two unobserved productivities (φ, ω) as functions of observables. This allows us to pin down the composite error η^* in the first stage, which can then be treated as "observable" in the subsequent steps to facilitate parameter identification. This is analogous to the Ackerberg et al. (2015) first step. In the next stage, we exploit the Markovian structure assumed for the labor-saving productivity φ and apply dynamic panel methods to the firm's optimality conditions adjusted for the error η^* to identify parameters related to the output elasticities of variable inputs. Production-function parameters related to the fixed input are identified in the last stage akin to the Gandhi et al. (2020) second step, whereby we use the Markovian process assumed for Hicksian ω to transform the latter out of the model in the tradition of Blundell and Bond (2000).

Stage One.—Akin to the initial step in the Ackerberg et al. (2015) methodology, we first isolate the noise η^* by estimating the so-called reduced-form revenue production function. A firm's FOCs for the short-run optimization (3.11) imply implicit (non-stochastic) functions for its optimal variable input choices expressed in terms of state variables $\left(\varphi, \omega, K, \chi', P^M, P^L, \overline{P}, \overline{Y}\right)'$. Take the materials. Using log-inputs instead of levels, fundamentally we have that $m = h(\varphi, \omega, k, \chi)$, where we have suppressed common determinants $\left(P^M, P^L, \overline{P}, \overline{Y}\right)'$ which do not vary across firms.

Assumption 4 *Conditional material demand is strictly increasing in factor-neutral ω.*

Making a routine monotonicity assumption about the short-run demand for materials, whereby more Hicks-neutrally productive firms use more materials,[16] we can invert the conditional material demand $h(\cdot)$ to express ω as $\omega = h^{-1}(k, m, \chi, \varphi)$. Using this proxy to substitute for ω and the proxy (3.4) to substitute for φ in the revenue production function (3.13) above, we arrive at its "reduced form" as an unspecified function $\Phi(\cdot)$, viz.,

$$\widetilde{r} = \Phi\left(k, l, m, \frac{P^L L}{P^M M}, \chi\right) + \eta^*, \tag{3.14}$$

in which no individual component of the revenue production function is separately identifiable due to functional dependence among regressors following the introduction of a nonparametric component

[16]The assumption was introduced by Levinsohn and Petrin (2003) in the context of neutral production technology and perfect competition but later also adapted to models with market power as well as non-neutral productivity (see De Loecker et al., 2016; Zhao et al., 2020; Malikov et al., 2024; Demirer, 2025).

$h^{-1}(\cdot)$ and where, like earlier, we have suppressed firm-invariant covariates $\left(P^M, P^L, \overline{P}, \overline{Y}\right)'$.

Albeit the reduced form in (3.14) does not directly aid the identification of any structural parameter, it however nonparametrically identifies a firm-specific composite shock η^* as a residual based on the following moment conditions:

$$\mathbb{E}\left[\eta^* \,\middle|\, k, l, m, \tfrac{P^L L}{P^M M}, \chi\right] = 0 \quad \text{and} \quad \mathbb{E}\left[\eta^*\right] = 0. \tag{3.15}$$

We are thus able to recover η^* and, as such, following this stage it becomes "known" and so does its mean $\Lambda^* = \mathbb{E}\left(e^{\eta^*}\right)$.

Stage Two.—Under the translog specification, the explicit parametric form of the share equations in (3.12) is given by

$$S^L = \left(\beta_l^* + \beta_{kl}^* k + \beta_{lm}^* m + \beta_{ll}^* l + \beta_{ll}^* \varphi\right) \Lambda^* e^{-\eta^*}, \tag{3.16}$$

$$S^M = \left(\beta_m^* + \beta_{km}^* k + \beta_{mm}^* m + \beta_{lm}^* l + \beta_{lm}^* \varphi\right) \Lambda^* e^{-\eta^*}. \tag{3.17}$$

Using the already identified η^* and Λ^* from the first stage, these share equations yield two deterministic proxy expressions for φ each one of which, when combined with the law of motion for the latter, helps identify subsets of the production-function parameters.

Concretely, start with the labor share equation (3.16). Solving it for φ, we have

$$\varphi = \frac{1}{\beta_{ll}^*} S^{L*} - \frac{\beta_l^*}{\beta_{ll}^*} - \frac{\beta_{kl}^*}{\beta_{ll}^*} k - \frac{\beta_{lm}^*}{\beta_{ll}^*} m - l,$$

where the rescaled $S^{L*} \equiv S^L e^{\eta^*}/\Lambda^*$ is now known. Replacing φ in the law of motion (3.7) with the above share-based proxy expression, after trivial manipulations we obtain a linear quasi-difference panel-data model:

$$\begin{aligned} S^{L*} = {} & \beta_l^*(1-\alpha_1) + \alpha_1 \mathbb{L}\left[S^{L*}\right] + \\ & \beta_{kl}^*\left(k - \alpha_1 \mathbb{L}[k]\right) + \beta_{ll}^*\left(l - \alpha_1 \mathbb{L}[l]\right) + \beta_{lm}^*\left(m - \alpha_1 \mathbb{L}[m]\right) + \alpha_2 \beta_{ll}^* \mathbb{L}[Z_1] + \epsilon^*, \end{aligned} \tag{3.18}$$

where $\epsilon^* \equiv \beta_{ll}^* \epsilon$. Unless inputs k, l or m are autoregressive (AR) of order 1 with the AR coefficient being exactly α_1, in which case there would be no informative variation to identify the slopes $\left(\beta_{kl}^*, \beta_{lm}^*, \beta_{ll}^*\right)'$, all unknown parameters in (3.18) can be point-identified based on nonlinear moments using predetermined and lagged inputs as instruments under Assumption 2:

$$\mathbb{E}\left[\epsilon^* \,\middle|\, \mathbb{L}\left[S^{L*}\right], k, \mathbb{L}[k], \mathbb{L}[m], \mathbb{L}[l], \mathbb{L}[Z_1]\right] = 0 \quad \text{and} \quad \mathbb{E}\left[\epsilon^*\right] = 0, \tag{3.19}$$

as the innovation ϵ is orthogonal to variables in the previous-period information set. Fortunately, per the underlying structural model, inputs are unlikely to follow such a simple AR process. Also note that the contemporaneous and endogenous l and m require no additional instrumentation as they enter (3.18) subject to parameter restrictions. With this, we have identified $(\alpha_1, \alpha_2)'$ and all revenue labor-elasticity coefficients $\left(\beta_l^*, \beta_{ll}^*, \beta_{kl}^*, \beta_{lm}^*\right)'$.

To identify revenue material-elasticity parameters that remain unknown $\left(\beta_m^*, \beta_{mm}^*, \beta_{km}^*\right)'$, we make use of the second share equation in (3.17) following an analogous procedure. Specifically, solving (3.17) for φ, we have an alternative proxy expression:

$$\varphi = \frac{1}{\beta_{lm}^*} S^{M*} - \frac{\beta_m^*}{\beta_{lm}^*} - \frac{\beta_{km}^*}{\beta_{lm}^*} k - \frac{\beta_{mm}^*}{\beta_{lm}^*} m - l,$$

where $S^{M*} \equiv S^M e^{\eta^*} / \Lambda^*$ is known just like S^{L*}. Combining with (3.7) and letting $\epsilon_{it}^{\dagger} \equiv \beta_{lm}^* \epsilon_{it}$, we arrive at

$$\begin{aligned} S^{M*} = {} & \beta_m^*(1-\alpha_1) + \alpha_1 \mathbb{L}\left[S^{M*}\right] + \\ & \beta_{km}^*\left(k - \alpha_1 \mathbb{L}[k]\right) + \beta_{mm}^*\left(m - \alpha_1 \mathbb{L}[m]\right) + \beta_{lm}^*\left(l - \alpha_1 \mathbb{L}[l]\right) + \alpha_2 \beta_{lm}^* \mathbb{L}[Z_1] + \epsilon^{\dagger}, \end{aligned}$$

which is akin to (3.18) and where the parameters $(\alpha_1, \alpha_2, \beta_{lm}^*)'$ have already been identified. Treating those as known, we rewrite the above equation as follows to isolate yet-to-be-identified $\left(\beta_m^*, \beta_{mm}^*, \beta_{km}^*\right)'$ on the right-hand side:

$$\widetilde{S}^{M*} = \beta_m^*(1-\alpha_1) + \beta_{km}^* \triangle k + \beta_{mm}^* \triangle m + \epsilon^{\dagger}, \tag{3.20}$$

where $\widetilde{S}^{M*} \equiv \left(S^{M*} - \alpha_1 \mathbb{L}\left[S^{M*}\right]\right) - \beta_{lm}^*\left(l - \alpha_1 \mathbb{L}[l]\right) - \alpha_2 \beta_{lm}^* \mathbb{L}[Z_1]$, $\triangle k \equiv k - \alpha_1 \mathbb{L}[k]$ and $\triangle m \equiv m - \alpha_1 \mathbb{L}[m]$ are all known. This is a linear model with an endogenous $\triangle m$, and we can identify $(\beta_m^*, \beta_{mm}^*, \beta_{km}^*)'$ in it under Assumption 2 based on

$$\mathbb{E}\left[\epsilon^{\dagger}\left((1-\alpha_1), \triangle k, \mathbb{L}[m]\right)'\right] = 0, \tag{3.21}$$

where $\mathbb{L}[m]$ is an obvious choice of a valid instrument for $\triangle m$.

Note that all production-function parameters identified in the second stage are the "β^*" coefficients. To recover the underlying "β" parameters, we still need to identify the demand elasticity so that the former can be rescaled by the inverse of $\left(1 + \frac{1}{\sigma}\right)$. At the same time, following the second stage, we have effectively identified labor-augmenting φ as can be seen from any one of its proxy expressions that we have used. This is also clear from (3.4), whereby not knowing $\left(1 + \frac{1}{\sigma}\right)$ does not contaminate the identification of φ since it is a function of relative coefficients. Thus, the effective labor $\widetilde{l}$ is now known too.

Stage Three.—We next focus on the remaining unknown parameters in (3.13). Collecting together all known and already identified components of the revenue production function as $\widetilde{r}^* \equiv \widetilde{r} - \beta_l^* \widetilde{l} - \beta_m^* m - \beta_{kl}^* k \widetilde{l} - \beta_{km}^* km - \beta_{lm}^* \widetilde{l} m - \frac{1}{2}\beta_{ll}^* \widetilde{l}^2 - \frac{1}{2}\beta_{mm}^* m^2 - \eta^*$, we have $\widetilde{r}^* = \beta_k^* k + \frac{1}{2}\beta_{kk}^* k^2 + \left(1 + \frac{1}{\sigma}\right)\omega - \frac{1}{\sigma}\overline{y} + \rho'\chi$ from where we obtain an analytical deterministic proxy expression for ω:

$$\omega = \frac{\sigma}{1+\sigma}\widetilde{r}^* - \beta_k k - \tfrac{1}{2}\beta_{kk} k^2 + \frac{1}{1+\sigma}\overline{y} - \frac{\sigma}{1+\sigma}\rho'\chi. \tag{3.22}$$

Substituting it for ω in the law of motion (3.8), we get another linear quasi-difference panel-data model:

$$\widetilde{r}^* = \left(1+\tfrac{1}{\sigma}\right)\gamma_0 + \gamma_1\mathbb{L}\left[\widetilde{r}^*\right] + \\ \beta_k^*\left(k-\gamma_1\mathbb{L}[k]\right) + \tfrac{1}{2}\beta_{kk}^*\left(k^2-\gamma_1\mathbb{L}[k]^2\right) - \tfrac{1}{\sigma}\left(\overline{y}-\gamma_1\mathbb{L}[\overline{y}]\right) + \rho'\left(\chi-\gamma_1\mathbb{L}[\chi]\right) + \left(1+\tfrac{1}{\sigma}\right)\gamma_2\mathbb{L}[Z_2] + \varepsilon^*, \tag{3.23}$$

where $\varepsilon^* \equiv \left(1+\frac{1}{\sigma}\right)\varepsilon$. Under Assumption 2, we have the following moment conditions

$$\mathbb{E}\left[\varepsilon^* \middle| \mathbb{L}\left[\widetilde{r}^*\right], k, \mathbb{L}[k], k^2, \mathbb{L}[k]^2, \mathbb{L}[\overline{y}], \mathbb{L}[\chi], \mathbb{L}[Z_2]\right] = 0 \quad \text{and} \quad \mathbb{E}\left[\varepsilon^*\right] = 0, \tag{3.24}$$

through which we identify the rest of unknown parameters $\left(\sigma,\gamma_0,\gamma_1,\gamma_2,\beta_k^*,\beta_{kk}^*,\rho'\right)'$. With the demand elasticity σ now identified, we can recover all "β" estimates via $\beta = \beta^*/(1+\frac{1}{\sigma})$. Finally, from (3.22), we also readily have ω.

4 Estimator

Following the identification strategy fleshed out in the previous section, we can estimate the revenue production function in three sequential easy-to-implement steps. For clarity, let us introduce explicit firm ($i = 1,\ldots,N$) and time ($t = 1,\ldots,T$) subscripts.

Moment restrictions in (3.15) suggest nonparametric least squares (LS) for estimating the first-stage reduced form in (3.14) as all its regressors are contemporaneously exogenous with respect to a composite noise η^*. We approximate the unknown function $\Phi(\cdot)$ using standard second-order polynomial sieves, i.e., for a vector of J variables $x_{it} = (x_{1,it},\ldots,x_{J,it})'$:

$$\Phi(x_{it}) \approx \sum_{\kappa_1+\cdots+\kappa_J\leq 2} \theta_{\kappa_1,\ldots,\kappa_J} x_{1,it}^{\kappa_1}\cdots x_{J,it}^{\kappa_J} + \sum_{\tau\leq T}\theta_\tau D_{it}^\tau, \tag{4.1}$$

where $\kappa_j \in \{0,1,2\}$ for $j = 1,\ldots,J$, and $\{\theta_\tau\}$ are the time effects corresponding to year dummies $D_{it}^\tau \equiv \mathbb{1}\{t=\tau\}$ for $\tau = 1,\ldots,T$. In our case, $x_{it} = \left(k_{it}, l_{it}, m_{it}, \frac{P_t^L L_{it}}{P_t^M M_{it}}, \chi_{it}'\right)'$ and the time effects capture firm-invariant covariates $\left(P_t^M, P_t^L, \overline{P}_t, \overline{Y}_t\right)'$. Also, let all first-stage sieve "parameters" in (4.1) be collectively denoted as $\mathbf{\Theta}_1$. By minimizing $\sum_{i,t}\left[\eta_{it}^*(\mathbf{\Theta}_1)\right]^2$, where $\eta_{it}^*(\mathbf{\Theta}_1)$ is the difference between $\widetilde{r}_{it}$ and the $\Phi(\cdot)$ approximation above, which is equivalent to GMM based on

$$\mathbb{E}\left[\begin{bmatrix}\text{Polynomial}\left(k_{it}, l_{it}, m_{it}, \frac{P_t^L L_{it}}{P_t^M M_{it}}, \chi_{it}\right)' \\ \left(D_{it}^1,\ldots,D_{it}^T\right)'\end{bmatrix}\eta_{it}^*(\mathbf{\Theta}_1)\right] = \mathbf{0}, \tag{4.2}$$

we obtain consistent estimates of η_{it}^* in the form of residuals $\eta_{it}^*\big(\widehat{\mathbf{\Theta}}_1\big)$, using which we can also consistently estimate Λ^* as $\Lambda^*\big(\widehat{\mathbf{\Theta}}_1\big) = \frac{1}{NT}\sum_{i,t} e^{\eta_{it}^*(\widehat{\mathbf{\Theta}}_1)}$, readily providing adjusted shares $S_{it}^{L*}\big(\widehat{\mathbf{\Theta}}_1\big) \equiv S_{it}^L e^{\eta_{it}^*(\widehat{\mathbf{\Theta}}_1)}/\Lambda^*\big(\widehat{\mathbf{\Theta}}_1\big)$ and $S_{it}^{M*}\big(\widehat{\mathbf{\Theta}}_1\big) \equiv S_{it}^M e^{\eta_{it}^*(\widehat{\mathbf{\Theta}}_1)}/\Lambda^*\big(\widehat{\mathbf{\Theta}}_1\big)$ for the next stage.

The subsequent stage-two equations—namely, eqs. (3.18) and (3.20)—as well as the stage-three equation in (3.23) include contemporaneously endogenous covariates needing instrumentation but all are estimatable using the usual generalized method of moments (GMM) based on unconditional versions of identifying moment restrictions in (3.19), (3.21) and (3.24), with the use of intermediate "between-stage" estimators such as those of $\widetilde{S}^M_{it}$, $\triangle k_{it}$, $\triangle m_{it}$, $\widetilde{l}_{it}$ and $\widetilde{r}^*_{it}$. The recursive moments for these GMM estimators are

$$\mathbb{E}\left[\begin{bmatrix} S^{L*}_{it-1}(\widehat{\mathbf{\Theta}}_1) \\ \left(1, k_{it}, k_{it-1}, m_{it-1}, l_{it-1}, Z_{1,it-1}\right)' \end{bmatrix} \epsilon^*_{it}\Big(\overbrace{\beta^*_l, \beta^*_{kl}, \beta^*_{ll}, \beta^*_{lm}, \alpha_1, \alpha_2}^{\mathbf{\Theta}_{2\mathrm{L}}}, \widehat{\mathbf{\Theta}}_1\Big)\right] = \mathbf{0}, \quad (4.3)$$

$$\mathbb{E}\left[\begin{bmatrix} 1 - \widehat{\alpha}_1 \\ k_{it} - \widehat{\alpha}_1 k_{it-1} \\ m_{it-1} \end{bmatrix} \epsilon^\dagger_{it}\Big(\overbrace{\beta^*_m, \beta^*_{mm}, \beta^*_{km}}^{\mathbf{\Theta}_{2\mathrm{M}}}, \widehat{\mathbf{\Theta}}_{2\mathrm{L}}, \widehat{\mathbf{\Theta}}_1\Big)\right] = \mathbf{0}, \quad (4.4)$$

$$\mathbb{E}\left[\begin{bmatrix} \widetilde{r}^*_{it-1}(\widehat{\mathbf{\Theta}}_1, \widehat{\mathbf{\Theta}}_{2\mathrm{L}}, \widehat{\mathbf{\Theta}}_{2\mathrm{M}}) \\ \left(1, k_{it}, k_{it-1}, k^2_{it}, k^2_{it-1}, \overline{y}_{t-1}, \chi'_{it-1}, Z_{2,it-1}\right)' \end{bmatrix} \varepsilon^*_{it}\Big(\overbrace{\gamma_0, \gamma_1, \gamma_2, \beta^*_k, \beta^*_{kk}, \sigma, \rho'}^{\mathbf{\Theta}_3}, \widehat{\mathbf{\Theta}}_{2\mathrm{M}}, \widehat{\mathbf{\Theta}}_{2\mathrm{L}}, \widehat{\mathbf{\Theta}}_1\Big)\right] = \mathbf{0}, \quad (4.5)$$

where $\epsilon^*_{it}(\cdot)$, $\epsilon^\dagger_{it}(\cdot)$ and $\varepsilon^*_{it}(\cdot)$ are residual functions along with their corresponding unknown parameter sets $\mathbf{\Theta}_{2\mathrm{L}}$, $\mathbf{\Theta}_{2\mathrm{M}}$ and $\mathbf{\Theta}_3$ from (3.18), (3.20) and (3.23), respectively. Also, recall that, once $\widehat{\sigma}$ is obtained in the last step, we can consistently recover all unscaled "$\widehat{\beta}$" estimates via $\widehat{\beta} = \widehat{\beta}^* / \left(1 + \frac{1}{\widehat{\sigma}}\right)$.

Under our model assumptions, such a multi-step estimator is consistent and normal in the limit as N diverges to ∞. This follows by recognizing that our estimator falls within Ai and Chen's (2003) general framework for semiparametric sieve minimum distance m-estimators based on moment restrictions, allowing us to extend their large-sample results to here. To establish this, we can recast all our estimation steps as a simultaneous moment-based recursive GMM problem, where instrument sets (some of which are nonlinear functions of parameters themselves) vary across equations and there exist cross-equation parameter restrictions, yielding an Ai and Chen's (2003)-type multi-equation moment restriction of a generic form $\mathbb{E}\left[\varrho\big(\Phi(\cdot), \mathbf{\Theta}_{2\mathrm{L}}, \mathbf{\Theta}_{2\mathrm{M}}, \mathbf{\Theta}_3, \mathbb{X}\big)\mathbb{Q}\right] = \mathbf{0}$, where $\varrho(\cdot)$ is a vector of residual functions from all estimation stages, $\mathbb{X}$ and $\mathbb{Q}$ are some data, the reduced-form production function $\Phi(\cdot)$ is an infinite-dimensional nonparametric function with the corresponding sieve "parameters" $\mathbf{\Theta}_1$, and $\left(\mathbf{\Theta}_{2\mathrm{L}}, \mathbf{\Theta}_{2\mathrm{M}}, \mathbf{\Theta}_3\right)'$ are the unknown parameters of fixed dimension.

If desired, such a system-estimator recasting also permits obtaining an asymptotic variance for the multi-stage estimator that "corrects" for its sequential implementation (see Newey, 1984; Ackerberg et al., 2012; Hahn et al., 2018). Having said that, should one find evaluating the analytical covariance matrix corresponding to multi-equation moment restrictions tedious or in the case when asymptotic inference is difficult to justify, bootstrap provides an alternative avenue for hypothesis testing. The latter is the route that we pursue, approximating the sampling distribution of our estimator via nonparametric block bootstrap (resampling cross-sections) that takes into account its sequential nature.

5 Monte-Carlo Simulations

Before applying our methodology to real data, we conduct a series of Monte Carlo simulation exercises to (i) evaluate the performance of the proposed estimator in recovering the true not-homothetically-separable production technology, and (ii) assess the flexibility of the log-quadratic translog specification in approximating alternative, potentially misspecified, production technologies.

5.1 Finite-Sample Performance under Not-Homothetically-Separable Translog

We first evaluate the ability of our methodology to identify parameters of a full-translog production technology in finite samples. We also contrast its performance with that of more restrictive estimators that impose (homothetic) separability.

Our data generation process (DGP) is largely based on setups in Grieco et al. (2016), Gandhi et al. (2020), and Malikov et al. (2024). Specifically, we consider a balanced panel of $N = \{200, 400, 800\}$ firms operating during $T = 10$ time periods. Each panel is simulated 500 times. The true values of production-function parameters in (3.1) are $\beta_k = 0.2$, $\beta_{kk} = -0.01$, $\beta_l = 0.25$, $\beta_{ll} = 0.06$, $\beta_m = 0.5$, $\beta_{mm} = 0.04$, $\beta_{kl} = 0.02$, $\beta_{km} = 0.02$, and $\beta_{lm} = -0.05$. With this parameterization, the production technology is not homothetically separable in variable inputs.[17] The transitory output error is generated as $\eta_{it} \overset{\text{iid}}{\sim} \mathbb{N}(0, \sigma_\eta^2)$ with $\sigma_\eta = 0.07$. In the output demand function (3.9), we set the price elasticity $\sigma = -5$ and eliminate the observable demand shock χ_{it} to keep things simple; the unobserved demand shock is generated as $\xi_{it} \overset{\text{iid}}{\sim} \mathbb{N}(0, \sigma_\xi^2)$ with $\sigma_\xi = 0.05$. The two productivity components are generated for each firm as $\varphi_{it} = \alpha_1\varphi_{it-1} + \epsilon_{it}$ and $\omega_{it} = \gamma_0 + \gamma_1\omega_{it-1} + \varepsilon_{it}$, where $\alpha_1 = 0.5$, $\gamma_0 = 0.2$, $\gamma_1 = 0.6$, $\epsilon_{it} \overset{\text{iid}}{\sim} \mathbb{N}(0, \sigma_\varphi^2)$, and $\varepsilon_{it} \overset{\text{iid}}{\sim} \mathbb{N}(0, \sigma_\omega^2)$ with $\sigma_\omega = \sigma_\varphi = 0.1$. Their initial values are generated as $\varphi_{i0} \overset{\text{iid}}{\sim} \mathbb{U}(-1, 1)$ and $\omega_{i0} \overset{\text{iid}}{\sim} \mathbb{U}(-1, 1)$. We abstract away from productivity shifters for simplicity.

Capital K_{it} has the evolution process $K_{it} = I_{it-1} + (1-\delta_i)K_{it-1}$, where the firm-specific depreciation rate δ_i is uniformaly drawn from $\{0.05, 0.075, 0.10, 0.125, 0.15\}$, and the investment function I_{it-1} takes the form $I_{it-1} = K_{it-1}^{\iota_1}[\exp\{\omega_{it-1}\}]^{\iota_2}\left[\exp\left\{\varphi_{it-1}\right\}\right]^{\iota_3}$, where $\iota_1 = 0.8$ and $\iota_2 = \iota_3 = 0.1$. The initial level of capital is generated as $K_{i0} \overset{\text{iid}}{\sim} \mathbb{U}(10, 200)$. The optimal labor and materials series are generated by numerically solving the firm's FOCs in (3.12) after having already generated the series of $(K_{it}, \omega_{it}, \varphi_{it})$ for each firm and time period. Finally, we let $P_t^M = P_t^L = \frac{P_t}{Y_t^{1/\delta}}\Lambda^*$ and $P_t = 1$ for all t to simplify the calculation. In the estimation, the aggregate demand shifter $Y_t = \frac{1}{N}\sum_i Y_{it}$ is modeled using a simple average of all firms' output in period t.

Tables 1–3 summarize the simulation results. Table 1 reports cross-simulation mean estimates (Mean) and root mean squared errors (RMSE) for all production function, demand function, and productivity process parameters. Because the translog specification implies observation-specific output elasticities and RTS, we summarize their estimates within each replication using medians to miti-

[17] Given the DGPs for production variables we use, the choice of these β coefficient values also guarantees that monotonicity and curvature properties of the production function are satisfied virtually always in the generated data. The share of observations that violate input (curvature) monotonicity is, on average, about 0.1% (1.2%) across simulation replications.

Table 1: Simulation Results for Production Function, Demand, and Productivity Parameters

		Our Methodology						**Imposing Separability w/o Homogeneity**						**Imposing Homothetic Separability**					
		N=200		N=400		N=800		N=200		N=400		N=800		N=200		N=400		N=800	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
β_k	0.200	0.174	0.176	0.190	0.120	0.199	0.027	0.050	0.177	0.069	0.149	0.084	0.127	–0.148	0.351	–0.156	0.357	–0.158	0.358
β_l	0.250	0.256	0.025	0.252	0.018	0.251	0.012	0.349	0.102	0.355	0.107	0.359	0.110	0.485	0.242	0.485	0.239	0.481	0.232
β_m	0.500	0.495	0.027	0.498	0.018	0.499	0.011	0.561	0.068	0.556	0.061	0.550	0.053	0.574	0.079	0.578	0.081	0.582	0.082
β_{kl}	0.020	0.019	0.005	0.020	0.003	0.020	0.002	0.000	0.020	0.000	0.020	0.000	0.020	0.000	0.020	0.000	0.020	0.000	0.020
β_{km}	0.020	0.021	0.005	0.020	0.003	0.020	0.002	0.000	0.020	0.000	0.020	0.000	0.020	0.000	0.020	0.000	0.020	0.000	0.020
β_{lm}	–0.050	–0.052	0.020	–0.051	0.011	–0.050	0.005	–0.058	0.029	–0.064	0.027	–0.069	0.026	–0.086	0.048	–0.080	0.037	–0.076	0.027
β_{kk}	–0.010	–0.004	0.041	–0.008	0.028	–0.010	0.006	0.025	0.041	0.021	0.035	0.018	0.030	0.035	0.047	0.035	0.046	0.036	0.046
β_{ll}	0.060	0.062	0.015	0.061	0.009	0.061	0.005	0.088	0.036	0.093	0.038	0.097	0.040	0.086	0.041	0.080	0.030	0.076	0.018
β_{mm}	0.040	0.040	0.025	0.040	0.013	0.041	0.006	0.076	0.049	0.084	0.052	0.090	0.054	0.086	0.055	0.080	0.046	0.076	0.037
σ	–5.000	–5.069	0.665	–5.014	0.234	–5.012	0.128	–5.117	0.370	–5.063	0.232	–5.052	0.153	–5.013	0.223	–5.006	0.164	–4.978	0.090
α_1	0.500	0.504	0.045	0.501	0.029	0.499	0.014	0.596	0.096	0.597	0.097	0.598	0.099	0.911	0.425	0.929	0.437	0.943	0.445
γ_0	0.200	0.218	0.116	0.206	0.079	0.202	0.022	0.347	0.162	0.333	0.142	0.323	0.129	0.874	0.765	0.889	0.749	0.872	0.688
γ_1	0.600	0.600	0.014	0.600	0.010	0.600	0.007	0.582	0.024	0.580	0.023	0.578	0.023	0.579	0.051	0.579	0.046	0.579	0.035

Note: "Mean" and "RMSE" stand for the mean estimate and the root mean squared error, respectively.

Table 2: Simulation Results for Output Elasticities and RTS

		Our Methodology						**Imposing Separability w/o Homogeneity**						**Imposing Homothetic Separability**					
		N=200		N=400		N=800		N=200		N=400		N=800		N=200		N=400		N=800	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
K	0.282	0.284	0.049	0.283	0.036	0.280	0.014	0.192	0.093	0.188	0.096	0.184	0.098	0.052	0.247	0.044	0.247	0.045	0.239
L	0.322	0.322	0.005	0.322	0.004	0.322	0.002	0.319	0.009	0.319	0.008	0.319	0.007	0.355	0.039	0.357	0.037	0.356	0.033
M	0.630	0.631	0.009	0.631	0.006	0.630	0.004	0.623	0.013	0.624	0.010	0.624	0.009	0.704	0.077	0.707	0.073	0.706	0.066
RTS	1.227	1.228	0.053	1.228	0.039	1.226	0.015	1.123	0.112	1.122	0.114	1.119	0.117	1.096	0.143	1.093	0.145	1.092	0.145

Note: "True" and "Mean" denote the cross-simulation average of, respectively, the true and the estimated median elasticity. 'RMSE" denotes the cross-simulation root mean squared error of the median elasticity. The average *true* elasticities vary negligibly across sample sizes, with differences on the order of 0.001 in the first three digits after the decimal point.

Table 3: Simulation Results for Elasticities of Substitution

		Our Methodology						**Imposing Separability w/o Homogeneity**						**Imposing Homothetic Separability**					
		N=200		N=400		N=800		N=200		N=400		N=800		N=200		N=400		N=800	
	True	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
PANEL A: ALLEN-UZAWA ELASTICITY OF SUBSTITUTION																			
KL	0.819	0.852	0.137	0.830	0.076	0.824	0.039	1.186	0.380	1.168	0.358	1.152	0.338	1.287	0.633	1.267	0.668	1.308	0.714
KM	0.938	0.934	0.071	0.936	0.041	0.937	0.026	1.087	0.165	1.075	0.148	1.063	0.132	1.287	0.551	1.267	0.595	1.308	0.638
LM	1.401	1.408	0.282	1.412	0.142	1.407	0.047	1.622	0.372	1.692	0.384	1.746	0.393	1.686	2.776	1.521	2.445	1.478	0.815
PANEL B: MORISHIMA ELASTICITY OF SUBSTITUTION																			
KL	0.884	0.898	0.079	0.888	0.049	0.886	0.024	1.134	0.263	1.118	0.243	1.104	0.225	1.287	0.586	1.267	0.626	1.308	0.671
LK	1.137	1.159	0.137	1.150	0.077	1.143	0.035	1.426	0.315	1.456	0.335	1.477	0.349	1.517	1.545	1.436	1.341	1.434	0.538
KM	0.903	0.909	0.061	0.904	0.039	0.903	0.022	1.116	0.227	1.102	0.209	1.089	0.192	1.287	0.574	1.267	0.615	1.308	0.660
MK	1.074	1.074	0.108	1.076	0.037	1.075	0.013	1.245	0.179	1.256	0.187	1.264	0.192	1.407	0.926	1.347	0.830	1.358	0.510
LM	1.309	1.325	0.230	1.323	0.116	1.315	0.043	1.556	0.339	1.610	0.359	1.652	0.373	1.651	2.276	1.524	1.999	1.490	0.669
ML	1.328	1.336	0.241	1.339	0.116	1.333	0.038	1.538	0.319	1.594	0.334	1.637	0.344	1.651	2.274	1.524	1.997	1.490	0.664

Note: "True" and "Mean" denote the cross-simulation average of, respectively, the true and the estimated median elasticity. 'RMSE" denotes the cross-simulation root mean squared error of the median elasticity. The average *true* elasticities vary negligibly across sample sizes, with differences on the order of 0.001 in the first three digits after the decimal point.

gate the potential influence of outliers; Table 2 reports cross-simulation statistics of these median estimates. Table 3 reports the same for the corresponding Allen-Uzawa and Morishima ES.

We compare the performance of three estimators. The first one is that of our proposed methodology based on a full translog specification of nonneutral production technology. The other two estimators are implemented under separability restrictions. One imposes just the separability between (L,M) and K—without homogeneity in (L,M)—by restricting $\beta_{kl} = \beta_{km} = 0$ during the estimation. Whereas, the other one imposes a stronger, more commonly used homothetic separability assumption by additionally forcing $\beta_{ll} = \beta_{mm} = -\beta_{lm}$ onto the translog production function, further implying homogeneity in (L,M) of an arbitrary degree $\beta_l + \beta_m$ as discussed earlier.

In all three tables, we find that across all sample sizes our estimator tends to exhibit minimal mean bias, as evidenced by the small differences between the estimated and true values, and a shrinking RMSE as the sample size increases, consistent with root-N convergence. This good finite-sample performance demonstrates how our methodology can recover the true non-separable production technology in the presence of nonneutral technical change. In contrast, the two alternative estimators, which wrongly impose the (homothetic) separability assumption, exhibit persistent and sometimes sizable biases in the estimates of both parameters and implied output elasticities, with no convergence in RMSE. These lead to a mismeasurement of RTS. For instance, for $N = 800$, the RMSE for RTS using our estimator is a meager 0.015 (relative to the true average of 1.227), whereas for the other two estimators under separability it ranges between 0.117 and 0.145. A similar pattern emerges for both Allen–Uzawa and Morishima ES. The two estimators that impose separability restrictions exhibit substantial and persistent biases across many elasticity measures. The distortions are particularly pronounced under homothetic separability, where both mean deviations and RMSEs remain large even as the sample size increases.

Taken together, these findings underscore the perils of erroneously imposing (homothetic) separability in empirical production analysis when the true technology is nonseparable, highlighting the advantage of our methodology.

5.2 Approximation Performance under Misspecification

We next examine how sensitive our estimator is to functional-form misspecification, given that it is based on a translog production function whose flexibility stems from its common interpretation as a second-order Taylor expansion in natural logs. Specifically, we replace the true translog technology in the DGP with the following alternatives: a CES production function, a Generalized Cobb-Douglas (GCD) production function, and a higher-order generalized translog (GTL) production function augmented with cubic terms. Across these designs, we keep the remainder of the simulation environment unchanged (demand, productivity processes, capital accumulation, and the generation of variable inputs via the firm's FOCs), while adjusting certain production parameters to ensure stable simulated output and input choices. Our exercise echoes early contributions, such as Wales (1977) and Guilkey and Lovell (1980), that studied the ability of translog form to reveal characteristics of different production technologies. Unlike these earlier studies, however, our estimator builds on the modern structural framework that explicitly incorporates firms' optimizing behavior.

Online Appendix O4 reports full DGP details and complete results. In summary, the translog specification performs well when the true technology deviates moderately from the log-quadratic class. Under the CES and GCD designs, RTS and most substitution elasticities are recovered with small bias and limited dispersion. Whereas, the GTL experiments illustrate some limits of the approximation, as sizable output effects induced by cubic terms in the DGP exceed what a log-quadratic translog can flexibly approximate. When higher-order curvature is mild or symmetric, performance remains stable; however, asymmetric cubic terms can generate sizable distortions in substitution elasticities. Overall, the simulation evidence suggests that our translog form is robust for RTS and MES under moderate misspecification, whereas AES is more sensitive to pronounced nonlinear departures from the log-quadratic technology.

6 Empirical Results

We apply our proposed estimation methodology to three widely used production datasets available in the public domain from China, Chile, and Colombia.[18] Following Gandhi et al. (2020), we focus on the five largest industries in each country during their respective sample periods, estimating our model for each sector separately to allow for parameter heterogeneity.[19] Our analysis is organized around the central question of whether the evidence from the data is consistent with the implications of homothetically separable production technologies. Since our translog-based production function formulation does not *a priori* impose homothetic separability but nests such a specification as a special case (the RTL), we can test these implications both directly and indirectly. The direct tests focus on the parameter restrictions implied by homothetic separability, while the indirect tests rely on the two propositions developed in Section 2. As a preview of the results, we find that data provide little evidence supporting homothetic separability and its implications for technology. Additional results regarding production function estimates, a comparison between our model and the restricted translog model, and productivity estimates are presented in Online Appendix O5.

6.1 Direct Tests of Parameter Restrictions

We begin by formally testing the parameter restrictions implied by homothetic separability within the translog specification. Per our discussion in Section 3, it has been prevalent in the literature to restrict the translog by setting (i) $\beta_{kl} = \beta_{km} = 0$ and (ii) $\beta_{mm} = \beta_{ll} = -\beta_{lm}$. The first restriction ensures *separability* in the partition $(K,(L,M))$, whereas the second restriction imposes that the

[18]The data are described in Appendix C.

[19]We follow De Loecker (2011) in constructing the log of aggregate demand shifter $\overline{Y}$ in (3.9) as the weighted average of log deflated firm revenues within each 4-digit sub-industry of each studied sector, with weights given by firms' market shares. This approach introduces group-level variation into the variable. We also dispense with the demand shifter χ due to the lack of a consistent and reliable measure across countries and industries.

Table 4: Homothetic Separability Parameter Restriction Tests

	China					Colombia					Chile				
	Food	*Textile*	*Chemic.*	*Mineral*	*Machin.*	*Food*	*Textile*	*Apparel*	*Wood*	*Metals*	*Food*	*Textile*	*Apparel*	*Wood*	*Metals*
Separable in $(K,(L,M))$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.393	0.021	0.000
Homogeneous in (L,M)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.113	0.002
Homothetically Separable	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.081	0.054	0.003

Notes: Reported are the p-values for bootstrap Wald tests of the following parameter restrictions on the translog production function in (3.1): separability in the partition $(K,(L,M))$: $\beta_{kl} = \beta_{km} = 0$; homogeneity in (L,M) conditional on K: $\beta_{ll} = \beta_{mm} = -\beta_{lm}$; homothetic separability: $\beta_{kl} = \beta_{km} = 0$ and $\beta_{ll} = \beta_{mm} = -\beta_{lm}$.

Table 5: Median Elasticities of Substitution Between Capital and Each Variable Input

	China					Colombia					Chile				
	Food	*Textile*	*Chemic.*	*Mineral*	*Machin.*	*Food*	*Textile*	*Apparel*	*Wood*	*Metals*	*Food*	*Textile*	*Apparel*	*Wood*	*Metals*
Allen-Uzawa ES b/w K and L	1.995	1.126	1.255	1.322	0.789	0.551	1.024	1.191	–0.029	1.727	1.408	1.357	2.928	2.645	2.606
Allen-Uzawa ES b/w K and M	1.263	1.338	1.453	1.249	1.587	1.566	1.870	2.166	1.992	1.476	1.831	1.694	2.024	1.470	2.693
difference	0.732	–0.212	–0.198	0.073	–0.798	–1.015	–0.845	–0.975	–2.021	0.251	–0.424	–0.338	0.904	1.175	–0.088
	(0.175)	(0.092)	(0.056)	(0.018)	(0.065)	(0.175)	(0.493)	(0.265)	(0.615)	(0.478)	(0.183)	(0.535)	(1.804)	(1.185)	(1.205)
Morishima ES b/w K and L	1.322	1.309	1.420	1.500	1.457	1.370	1.441	1.756	1.383	1.627	1.701	1.545	2.515	1.916	2.736
Morishima ES b/w K and M	1.249	1.319	1.438	1.460	1.504	1.508	1.704	1.959	1.649	1.560	1.767	1.621	2.150	1.601	2.711
difference	0.073	–0.010	–0.018	0.040	–0.047	–0.139	–0.263	–0.204	–0.266	0.067	–0.066	–0.076	0.364	0.315	0.025
	(0.018)	(0.002)	(0.004)	(0.006)	(0.007)	(0.021)	(0.161)	(0.049)	(0.115)	(0.138)	(0.030)	(0.160)	(0.625)	(0.283)	(0.337)

Notes: Reported ES estimates are computed based on a nonseparable full translog production function in (3.1). Bootstrap standard errors for differences in medians are in parentheses.

Table 6: Median Elasticities of Substitution Between Variable Inputs

	China					Colombia					Chile				
	Food	*Textile*	*Chemic.*	*Mineral*	*Machin.*	*Food*	*Textile*	*Apparel*	*Wood*	*Metals*	*Food*	*Textile*	*Apparel*	*Wood*	*Metals*
Allen-Uzawa ES b/w L and M	–0.530	1.694	1.333	1.591	1.779	–0.351	3.417	–0.284	–0.795	3.452	2.091	2.093	–2.884	–2.740	–1.896
sample split: <0 / >0	55/45	39/61	47/53	38/62	40/60	55/45	44/56	51/49	58/42	35/65	24/76	32/68	94/6	96/4	90/10
Morishima ES b/w L and M	–0.249	1.657	1.325	1.623	1.682	–0.181	2.86	–0.030	–0.703	3.028	2.002	1.902	–1.530	–1.565	–0.668
sample split: <0 / >0	55/45	39/61	47/53	38/62	40/60	55/45	43/57	50/50	58/42	35/65	24/76	33/67	95/5	96/4	95/5
Morishima ES b/w M and L	–0.346	1.673	1.353	1.584	1.753	0.048	3.159	0.370	–0.363	2.988	2.098	2.032	–1.733	–1.837	–0.737
sample split: <0 / >0	55/45	39/61	47/53	37/63	40/60	47/53	42/58	49/51	57/43	34/66	24/76	30/70	92/8	97/3	85/15

Notes: Reported ES estimates are computed based on nonseparable full translog production function in (3.1). Sample splits of negative and positive point estimates are in percent.

technology is *homogeneous* in (L, M) (conditionally on K) of arbitrary degree $\beta_l + \beta_m$. Together, they give "homothetic separability." We test each one of these parameter restrictions as well as both of them together. Table 4 reports p-values from these tests. Across all industries in China and Colombia, with $p < 10^{-3}$ we strongly reject the null hypotheses of separability between K and (L, M), of homogeneity in (L, M) and, jointly, of homothetic separability. This suggests that, assuming a translog class of production functions, homothetic separability lacks support in these datasets. The restrictions are also rejected by the data in three of the five industries in Chile. The two exceptions are the apparel and wood industries, for which we reject—at the conventional 5% level—only one of the three nulls: the homogeneity ($p = 0.042$) for the apparel sector and the separability ($p = 0.021$) for the wood sector. These mixed results for Chile suggest that the homothetic separability assumption need not be empirically invalid in every single instance. Although a small minority of industries exhibit some evidence consistent with homothetic separability, the pattern is far from systematic. On the whole, we find that it is largely uncorroborated by the data, offering limited justification for its often summary imposition in applied work.

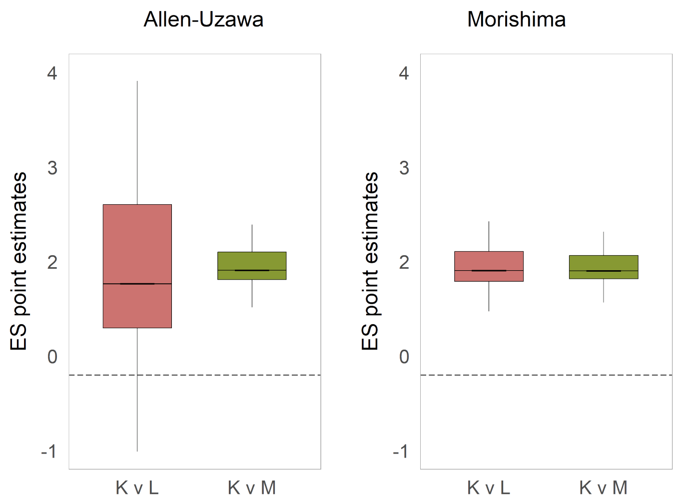
6.2 Indirect Evidence Based on Substitution and Scale Elasticity Structure

In Propositions 1 and 2, we derive technological and behavioral restrictions on input substitution structure and scale elasticity implied by the assumption of homothetic separability. These restrictions can be taken to the data and tested empirically (under our structural assumptions), thereby providing indirect evidence on whether production technologies are consistent with homothetic separability. Specifically, using the Allen-Uzawa and Morishima ES and the RTS calculated from our estimated not-homothetically-separable full translog production function, we examine whether:

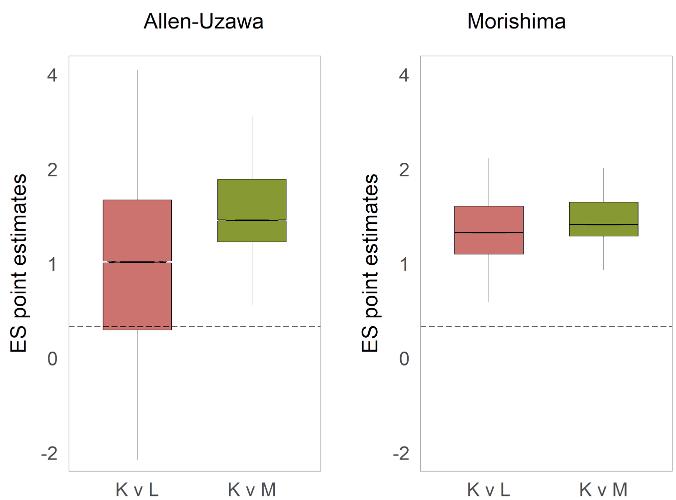
- ES between capital and each one of the variable inputs are equal (Prop. 1);
- ES between variable inputs is invariant to the level of capital (Prop. 2a);
- RTS are invariant to the effective labor price (Prop. 2b).

EQUALITY OF ES BETWEEN K AND $(L, M)'$.— The translog production function delivers not only heterogeneous output elasticities (and hence, RTS) but also observation-specific ES estimates. In Table 5 we report median AES and MES between capital and each one of the variable inputs across industries and countries. Of interest is whether the ES between K and L is equal to the ES between K and M, whereby capital is substitutable with both variable inputs with equal ease, as would be required by a homothetically separable production technology (Proposition 1). The results mostly indicate to the contrary. Under the Allen-Uzawa definition, across many industries, especially in the case of Colombia, the degree of substitutability of K with L and that with M deviate considerably. In China, albeit the median differences are more modest, they also remain economically and statistically significant. But for the Chilean data, the two elasticities significantly differ only for the food industry. Our findings based on the Morishima ES are qualitatively similar, although the corresponding cross-elasticity median differences are of smaller magnitudes.

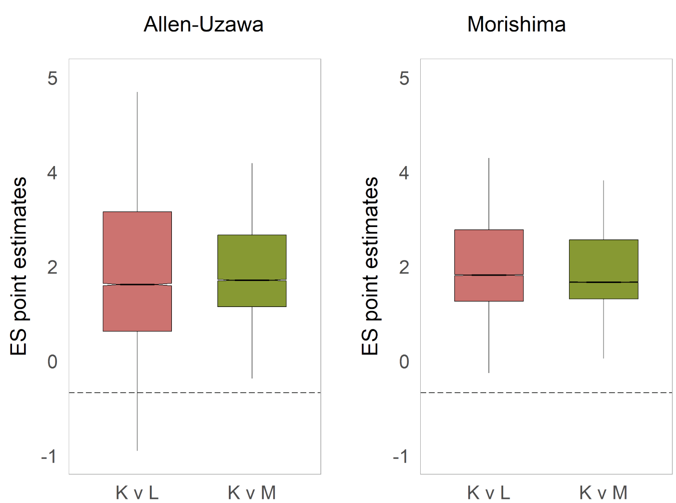
In addition to median values, in Figure 1 we also report boxplots of the ES between capital and variable inputs (pooled across industries). The distributions of the AES_{KL} and AES_{KM} are clearly



(a) CHINA



(b) COLOMBIA



(c) CHILE

Figure 1: Distributions of Elasticities of Substitution Between Capital and Each Variable Input

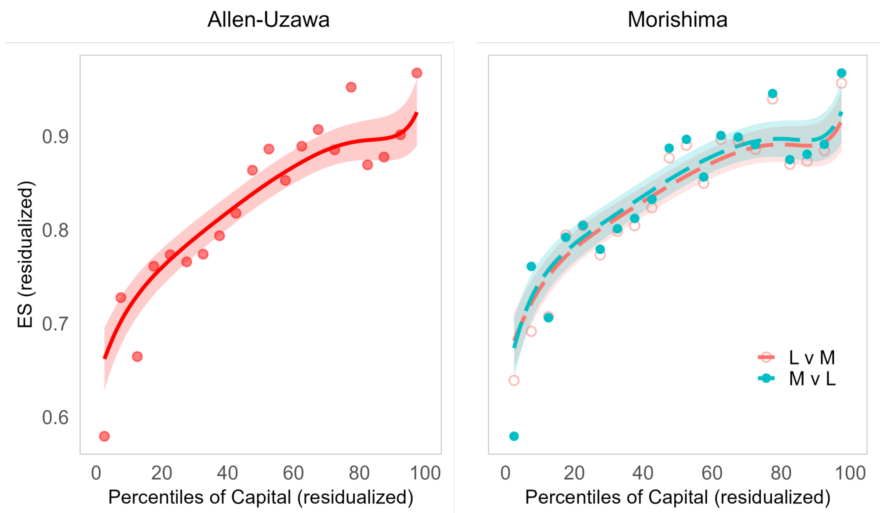
distinct, as reflected in their different box sizes, whisker lengths, and locations. Consistent with the evidence in Table 5, differences in the corresponding MES distributions are less visually pronounced, partly because the elasticities are more narrowly distributed. Beyond this, the visual evidence in Figure 1 underscores another notable point, namely, that substitution elasticities are non-constant and heterogeneous across firms which, e.g., the CES form *a priori* rules out.

INVARIANCE OF ES BETWEEN $(L,M)'$ WITH RESPECT TO K.— Per Proposition 2a, homothetic separability entails that the ES between variable inputs does not vary with the fixed level of capital. To examine this implication, Figure 2 plots the estimated AES and MES between labor and materials against the firm's capital, with all variables residualized by first regressing on industry fixed effects to remove between-industry variation for comparability.[20] Plotted are fitted kernel regression curves along with binned means of the variable-inputs ES by vigintile intervals of capital. The figure reveals a clear dependence on K of the ease of substitutability between variable inputs across all three countries, providing visual evidence against homothetic separability. The empirical relevance of capital for explaining the variation in variable-inputs ES is also buttressed by the cross-validated optimal bandwidths selected for plotted kernel regressions. As shown by Hall et al. (2007), local-constant kernel methods, which we employ here, can identify irrelevant regressors via cross-validation procedure: when optimally selected, a large bandwidth parameter would effectively "smooth out" the regressor, rendering the projection of ES onto K flat. This provides an indirect, data-driven method to assess if the ES between $(L,M)'$ is K-invariant. Across all countries and both ES definitions, the optimal bandwidth does not exceed 46.4% of the standard deviation of (residualized) capital, which averts smoothing out of K, thus rejecting invariance with respect to the latter. In China, the ES increases with capital, indicating that larger firms operate with higher substitutability between labor and materials. In Colombia and Chile, the opposite pattern emerges: substitution elasticities decrease with physical capital, suggesting that firms endowed with more capital experience greater complementarity or weaker substitutability between variable inputs.

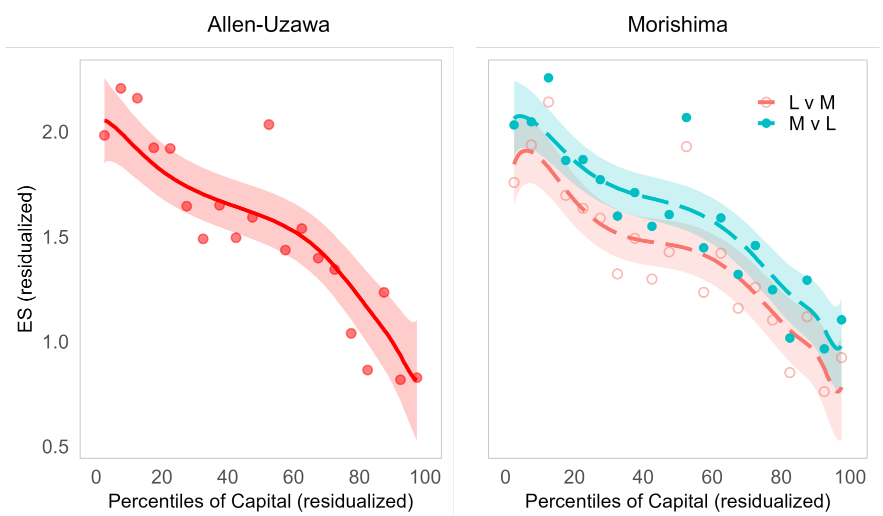
INVARIANCE OF RTS WITH RESPECT TO EFFECTIVE WAGE.— The last piece of indirect evidence we examine is the relation between firms' RTS and effective labor price. Per Proposition 2b, in response to the variation in productivity-adjusted labor price, which may stem from labor-augmenting productivity changes, the RTS does not vary when technology is homothetically separable. To this end, Figure 3 plots fitted kernel regressions of the RTS estimates on the computed effective wages $\widetilde{P}^L$ along with binned (by vigintile) means. All covariates here have been first residualized to partial out the industry effects and the effects of capital. The results reveal deviations from the theoretical prediction of price-invariance of the RTS, again providing little support for the homothetic separability restriction.[21] In China, the RTS decline over the lower-to-mid-wage range before leveling off. The relationship is non-monotonic in Colombia. Whereas, Chile shows the strongest negative relationship, with the RTS declining as effective wage increases, indicating that higher-effective-wage

[20] We have added a common pooled mean/intercept to the residualized ES before plotting.

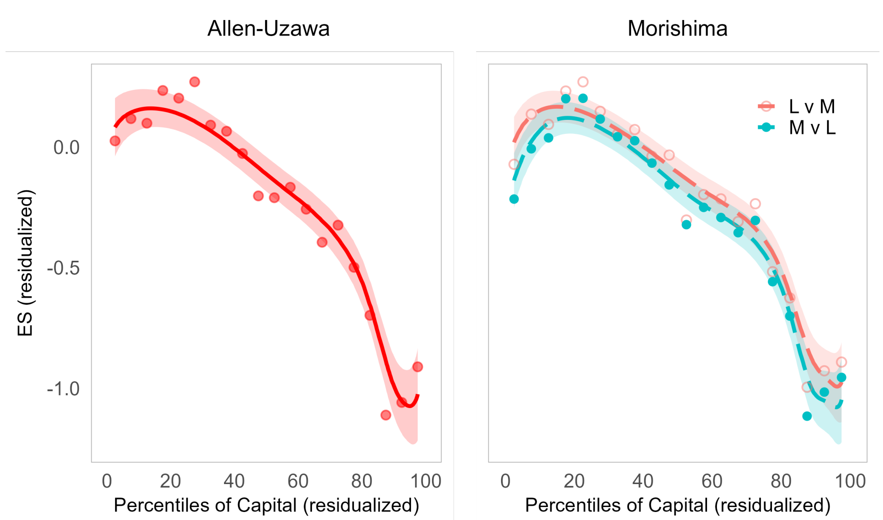
[21] All optimal bandwidths are smaller than 28.2% of the standard deviation of (residualized) effective wage, consistent with empirical relevance of the latter for the cross-firm variation in RTS.



(a) CHINA

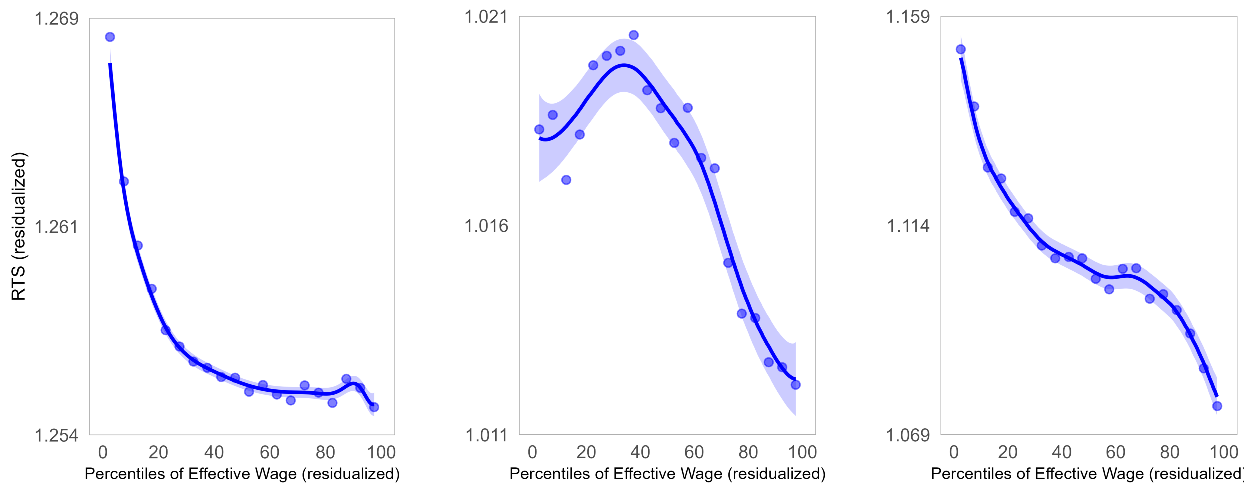


(b) COLOMBIA



(c) CHILE

Figure 2: Short-Run Scale Dependence of Elasticities of Substitution Between Variable Inputs



(a) CHINA (b) COLOMBIA (c) CHILE

Figure 3: Short-Run Dependence of Returns to Scale on the Effective Labor Price

firms tend to operate at smaller scale economies. Rationalizing these findings structurally (along with those of the nexus between variable-input substitutability and capital) is beyond the scope of our analysis since our focus is on testing homothetic separability, but it may prove to be a fruitful avenue for future research.

6.3 Additional Discussion

Beyond the implications derived from the homothetic separability assumption, we also examine two other restrictions on the production function often imposed in structural analyses of nonneutral technologies: i) the assumption of a constant—and homogeneous across firms—ES in CES-specification-based studies (Doraszelski and Jaumandreu, 2018), and ii) the restriction that variable inputs must be strictly either substitutes or complements in nonparametric formulations (Demirer, 2025).

Consider the degree of substitutability between the variable inputs L and M as estimated from our not-homothetically-separable model. Table 6 reports the associated median ES values, along with the sample breakdown (in %) for each industry-country pair based on the sign of the estimated observation-specific elasticities, offering insights into the predominance of complementarity (negative) versus substitutability (positive) between these inputs within the technology set. Our results reveal a substantial variation in substitutability between L and M across both countries and industries. For instance, in four of China's industries, median ES values are positive and above one, indicating strong substitutability between variable factors at the median. Of all industries, textiles is the only sector for which we find that L and M are substitutes in all three countries. In the case of the food industry, for example, the data suggest that labor and materials are weak complements in China and Colombia, whereas they are substitutes in Chile. More strikingly, the ease of substitution between L and M varies over the input space from net complementarity to net substitutability within the *same* industry-country pair—that is, given the same production function—as evidenced by negative-vs-positive-value sample splits, most of which are far from 0/100 or 100/0 by a lot. Not only does this ES heterogeneity disagrees with the CES specification but it also challenges the plausibility of assuming that variable inputs are strictly *either* substitutes *or* complements as required in a fully nonparametric approach.

Overall, we find that the data provide limited empirical imperative for imposing homothetic separability in structural productivity analyses, underscoring the value of using production function specifications that permit a more flexible (and heterogeneous) input substitution and RTS structure, such as the full translog. This finding is also reinforced by the recent empirical work by Hubmer et al. (2025) who provide extensive evidence against the homogeneity of production technologies, showing substantial RTS heterogeneity within industries across countries and data sources. That said, several caveats are warranted when interpreting the magnitude of the estimated elasticities and returns to scale. The structural framework adopted here imposes a number of assumptions, including monopolistic competition in output markets, competitive input markets with exogenous prices, and the absence of unobserved correlated demand heterogeneity. These restrictions on the model structure, while quite standard in the structural productivity analysis literature, may contribute to large magnitudes of ES (and sometimes RTS) in some industries and countries. The uncovered presence of strong input quantity responses to price changes should therefore be interpreted with caution. The magnitudes of estimated elasticities are to be viewed as conditional on our maintained structural assumptions rather than as primitives invariant to alternative market environments. Importantly, our objective here is to show that imposing homothetic separability can itself introduce distortions in empirical analyses and to demonstrate how the restrictions it implies can be tested empirically. In that sense, there is no need to begin from a specification that lacks empirical support.

7 Conclusion

This paper focuses on the homothetic separability restriction widely used in productivity analyses of firm-level gross production functions with multi-dimensional productivity, wherein technological change can manifest as both factor-neutral and labor-augmenting. The main takeaways are as follows. First, we show that homothetic separability in the fixed-vs-variable-factors partition entails restrictive economic implications for firm production technology and behavior that are neither trivial nor necessarily supported by the data. Second, for parametric production functions—here, the full translog—we demonstrate that homothetic separability is not a prerequisite for point-identifying the technology and recovering Hicksian and labor-augmenting productivity if one fully exploits the parameter restrictions on the firm's FOCs. Third, leveraging the variation in the optimal variable input levels rather than solely their ratios, we translate these insights into an easy-to-implement three-stage structural estimator situated within the canonical "proxy variable" framework. To maximize practical value of our methodology, we focus on the case when information on output quantities is unavailable with only revenues being reported in the data which, to this day, remains the most common occurrence with production datasets. Applying our methodology to three widely used production datasets from China, Colombia, and Chile, we find little evidence in support of imposing homothetic separability in most major industry. Altogether, our paper cautions against routine, often summary imposition of homothetic separability, illustrates the empirical relevance of production-function formulations with more flexible input substitution and RTS structures, and provides an estimator that applied researchers can implement with familiar proxy-variable tools.

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APPENDIX

A Proof of Proposition 1

Let a twice-continuously differentiable production function $F(K,e^{\varphi}L,M)$ be homothetically separable in $(e^{\varphi}L,M)$ from K per the restriction (2.2). We thus have $F\big(K,H(e^{\varphi}L,M)\big)$ where the subfunction $H(\cdot)$ is twice-continuously differentiable and homogeneous of an arbitrary degree α_H. To streamline the notation, denote $\Phi \equiv e^{\varphi}$ and, hence, the definition of effective labor is $\widetilde{L} \equiv e^{\varphi}L = \Phi L$.

From the primal definitions of AES and MES in (2.4)–(2.5), it follows that in order to show $\text{AES}_{KL} = \text{AES}_{KM}$ and $\text{MES}_{KL} = \text{MES}_{KM}$ we only need to prove that $\frac{\text{C}_{KL}}{L} = \frac{\text{C}_{KM}}{M}$, where C_{KL} and C_{KM} are cofactors of the bordered Hessian for the production function $F(\cdot)$.

Using the notation G_X, G_Z and G_{XZ} to denote first- and second-order partials of some function G with respect to its arguments, the bordered Hessian for a homothetically separable production function $F\big(K,H(\widetilde{L},M)\big)$ takes the following form

$$\begin{aligned}\overline{\text{B}} &= \begin{pmatrix} 0 & F_K & F_L & F_M \\ F_K & F_{KK} & F_{KL} & F_{KM} \\ F_L & F_{LK} & F_{LL} & F_{LM} \\ F_M & F_{MK} & F_{ML} & F_{MM} \end{pmatrix} \\ &= \begin{pmatrix} 0 & F_K & F_H(\Phi H_{\widetilde{L}}) & F_H H_M \\ F_K & F_{KK} & F_{KH}(\Phi H_{\widetilde{L}}) & F_{KH} H_M \\ F_H(\Phi H_{\widetilde{L}}) & F_{KH}(\Phi H_{\widetilde{L}}) & F_{HH}(\Phi H_{\widetilde{L}})^2 + F_H(\Phi^2 H_{\widetilde{L}\widetilde{L}}) & F_{HH}(\Phi H_{\widetilde{L}})H_M + F_H(\Phi H_{\widetilde{L}M}) \\ F_H H_M & F_{KH} H_M & F_{HH}(\Phi H_{\widetilde{L}})H_M + F_H(\Phi H_{\widetilde{L}M}) & F_{HH} H_M^2 + F_H H_{MM} \end{pmatrix}. \end{aligned} \tag{A.1}$$

The corresponding cofactors for the entries (K,L) and (K,M) are

$$C_{KL} = (-1)^{2+3} \det \begin{pmatrix} 0 & F_K & F_M \\ F_L & F_{LK} & F_{LM} \\ F_M & F_{MK} & F_{MM} \end{pmatrix}, \qquad C_{KM} = (-1)^{2+4} \det \begin{pmatrix} 0 & F_K & F_L \\ F_L & F_{LK} & F_{LL} \\ F_M & F_{MK} & F_{LM} \end{pmatrix},$$

and a straightforward expansion yields

$$C_{KL} = F_K F_H^2 \Phi\big(H_{\widetilde{L}} H_{MM} - H_M H_{\widetilde{L}M}\big) \quad \text{and} \quad C_{KM} = F_K F_H^2 \Phi^2\big(H_M H_{\widetilde{L}\widetilde{L}} - H_{\widetilde{L}} H_{\widetilde{L}M}\big). \tag{A.2}$$

We wish to prove that $\frac{C_{KL}}{L} - \frac{C_{KM}}{M} = 0$. From (A.2), we have

$$\frac{C_{KL}}{L} - \frac{C_{KM}}{M} = F_K F_H^2 \Phi^2 \left[\frac{H_{\widetilde{L}} H_{MM} - H_M H_{\widetilde{L}M}}{\widetilde{L}} - \frac{H_M H_{\widetilde{L}\widetilde{L}} - H_{\widetilde{L}} H_{\widetilde{L}M}}{M} \right], \tag{A.3}$$

so let us consider the difference in brackets.

Since $H(\cdot)$ is homogeneous of degree α_H in $\big(\widetilde{L},M\big)$, its partials are homogeneous of degree $\alpha_H - 1$.

Per Euler's theorem, we then have

$$\widetilde{L}H_{\widetilde{L}\widetilde{L}} + MH_{\widetilde{L}M} = (\alpha_H - 1)H_{\widetilde{L}} \quad \text{and} \quad \widetilde{L}H_{\widetilde{L}M} + MH_{MM} = (\alpha_H - 1)H_M,$$

from where

$$H_{\widetilde{L}\widetilde{L}} = \frac{(\alpha_H - 1)H_{\widetilde{L}} - MH_{\widetilde{L}M}}{\widetilde{L}}, \qquad H_{MM} = \frac{(\alpha_H - 1)H_M - \widetilde{L}H_{\widetilde{L}M}}{M}.$$

Substituting these expressions for $H_{\widetilde{L}\widetilde{L}}$ and H_{MM} in (A.3) reduces *both* fractions inside the brackets, $\frac{H_{\widetilde{L}}H_{MM} - H_M H_{\widetilde{L}M}}{\widetilde{L}}$ and $\frac{H_M H_{\widetilde{L}\widetilde{L}} - H_{\widetilde{L}} H_{\widetilde{L}M}}{M}$, to

$$\frac{(\alpha_H - 1)H_{\widetilde{L}}H_M}{M\widetilde{L}} - H_{\widetilde{L}M}\left(\frac{H_{\widetilde{L}}}{M} + \frac{H_M}{\widetilde{L}}\right),$$

with their difference thus being zero. Therefore, under the maintained assumption of strict monotonicity, ensuring that $F_K F_H^2 \neq 0$, it follows that $\frac{C_{KL}}{L} - \frac{C_{KM}}{M} = 0$, which completes the proof.

B Short-Run Production under Homothetic Separability

Consider a risk-neutral firm's ex-ante variable cost minimization problem "in the short run" necessary for (expected) restricted profit maximization:

$$\begin{aligned} &\min_{L,M} \; P^L L + P^M M: \quad \overbrace{e^{-\omega} Y^*/\Lambda}^{\widetilde{Y}^*} = F\left(K, e^{\varphi}L, M\right) \\ \Longleftrightarrow \; &\min_{\widetilde{L},M} \; \widetilde{P}^L \widetilde{L} + P^M M: \quad \widetilde{Y}^* = F\left(K, \widetilde{L}, M\right) \end{aligned} \tag{B.1}$$

where $\widetilde{Y}^*$ is the planned output level (given the demand-side factors, productivity as well as fixed K) net of factor-neutral productivity, with $Y^* \equiv \mathbb{E}[Y|\mathscr{I}]$ and $\Lambda \equiv \mathbb{E}[e^{\eta}|\mathscr{I}] = \mathbb{E}[e^{\eta}]$.

B.1 Variable cost function

In a general case with*out* homothetic separability, solving (B.1) yields a well-familiar value function—the variable cost function $VC\left(\widetilde{Y}^*, K, \widetilde{P}^L, P^M\right)$—which need not be functionally separable in any of its arguments. However, when $F\left(K, \widetilde{L}, M\right) = F\left(K, H(\widetilde{L}, M)\right)$ with $H(\cdot)$ being homogeneous, it is.

Namely, under (2.2), we can reformulate the firm's variable cost function corresponding to (B.1) using the homothetic "short-run production function" representation in (2.6) as follows:

$$
\begin{aligned}
VC\left(\widetilde{Y}^*,K,\widetilde{P}^L,P^M\right) &= \min_{\widetilde{L},M}\left\{\widetilde{P}^L\widetilde{L}+P^M M: \quad \widetilde{Y}^* \leq F\left(K,H(\widetilde{L},M)\right)\right\} \\
&= \min_{\widetilde{L},M}\left\{\widetilde{P}^L\widetilde{L}+P^M M: \quad F^{-1}\left(\widetilde{Y}^*,K\right) \leq H(\widetilde{L},M)\right\} \\
&= \min_{\widetilde{L},M}\left\{\widetilde{P}^L\widetilde{L}+P^M M: \quad 1 \leq \frac{1}{F^{-1}\left(\widetilde{Y}^*,K\right)}H(\widetilde{L},M)\right\} \\
&= \min_{\widetilde{L},M}\left\{\widetilde{P}^L\widetilde{L}+P^M M: \quad 1 \leq H\left(\left[\frac{1}{F^{-1}\left(\widetilde{Y}^*,K\right)}\right]^{\frac{1}{\alpha_H}}\widetilde{L},\ \left[\frac{1}{F^{-1}\left(\widetilde{Y}^*,K\right)}\right]^{\frac{1}{\alpha_H}}M\right)\right\},
\end{aligned}
$$

where the last equality follows from the homogeneity property of $H(\widetilde{L},M)$, which is homogeneous of degree α_H in $(\widetilde{L},M)$.

Define

$$
\widetilde{L}^* = \left[\frac{1}{F^{-1}\left(\widetilde{Y}^*,K\right)}\right]^{\frac{1}{\alpha_H}}\widetilde{L} \qquad \text{and} \qquad M^* = \left[\frac{1}{F^{-1}\left(\widetilde{Y}^*,K\right)}\right]^{\frac{1}{\alpha_H}}M.
$$

Then, we can rewrite the variable cost function as

$$
\begin{aligned}
VC\left(\widetilde{Y}^*,K,\widetilde{P}^L,P^M\right) &= \min_{\widetilde{L}^*,M^*}\left\{\left[F^{-1}\left(\widetilde{Y}^*,K\right)\right]^{\frac{1}{\alpha_H}}\left(\widetilde{P}^L\widetilde{L}^*+P^M M^*\right): \quad 1 \leq H\left(\widetilde{L}^*,M^*\right)\right\} \\
&= \left[F^{-1}\left(\widetilde{Y}^*,K\right)\right]^{\frac{1}{\alpha_H}} \min_{\widetilde{L}^*,M^*}\left\{\widetilde{P}^L\widetilde{L}^*+P^M M^*: \quad 1 \leq H\left(\widetilde{L}^*,M^*\right)\right\} \\
&\equiv \left[F^{-1}\left(\widetilde{Y}^*,K\right)\right]^{\frac{1}{\alpha_H}} VC^1\left(\widetilde{P}^L,P^M\right),
\end{aligned} \tag{B.2}
$$

where $VC^1\left(\widetilde{P}^L,P^M\right)$ is the unit variable cost function for the intermediate product $F^{-1}\left(\widetilde{Y}^*,K\right)$. This completes the derivation of (2.7).

B.2 Invariance of RTS to effective variable input prices

By Shephard's lemma, under the homothetic separability (2.2), the firm's optimal labor-to-materials ratio only depends on (productivity-adjusted) factor prices:

$$
\frac{\widetilde{L}}{M} = \frac{\frac{\partial}{\partial \widetilde{P}^L}VC_1\left(\widetilde{P}^L,P^M\right)}{\frac{\partial}{\partial P^M}VC_1\left(\widetilde{P}^L,P^M\right)} \quad \Leftrightarrow \quad \frac{L}{M} = \frac{\frac{\partial}{\partial P^L}VC_1\left(e^{-\varphi}P^L,P^M\right)}{\frac{\partial}{\partial P^M}VC_1\left(e^{-\varphi}P^L,P^M\right)}. \tag{B.3}
$$

An immediate implication is that the output and capital elasticities of the variable cost—viz., $\epsilon^Y \equiv \frac{\partial \ln VC}{\partial \ln \widetilde{Y}^*} = \frac{\partial \ln VC}{\partial \ln Y^*}$ and $\epsilon^K \equiv \frac{\partial \ln VC}{\partial \ln K}$, respectively—are both orthogonal to factor prices. Thus, in response to the variation in effective price of labor, from (B.2) we have that $\frac{\partial \ln \epsilon^Y}{\partial \ln \widetilde{P}^L} = 0$ and $\frac{\partial \ln(-\epsilon^K)}{\partial \ln \widetilde{P}^L} = 0$. In a general *non*homothetic case, however, these price elasticities of output/capital elasticities would be non-zero, endogenizing returns to scale:

$$
\begin{aligned}
\frac{\partial \ln \epsilon^Y}{\partial \ln \widetilde{P}^L} &= \frac{\partial \epsilon^Y}{\partial \ln \widetilde{P}^L}\frac{1}{\epsilon^Y} = \frac{\partial^2 \ln VC}{\partial \ln Y^* \left(\partial \ln \widetilde{P}^L\right)}\frac{1}{\epsilon^Y} = \frac{\partial \mathrm{VCS}^L}{\partial \ln Y^*}\frac{1}{\epsilon^Y} = \frac{\partial \ln \mathrm{VCS}^L}{\partial \ln Y^*}\frac{\mathrm{VCS}^L}{\epsilon^Y} \\
&= \frac{\partial \ln\left(\frac{P^L L}{P^M M}\right)}{\partial \ln Y^*} \times \frac{\left(1-\mathrm{VCS}^L\right)\mathrm{VCS}^L}{\epsilon^Y}, \qquad \text{(B.4)}\\
\frac{\partial \ln\left(-\epsilon^K\right)}{\partial \ln \widetilde{P}^L} &= \frac{\partial \epsilon^K}{\partial \ln \widetilde{P}^L}\frac{1}{\epsilon^K} = \frac{\partial^2 \ln VC}{\partial \ln K \left(\partial \ln \widetilde{P}^L\right)}\frac{1}{\epsilon^K} = \frac{\partial \mathrm{VCS}^L}{\partial \ln K}\frac{1}{\epsilon^K} = \frac{\partial \ln \mathrm{VCS}^L}{\partial \ln K}\frac{\mathrm{VCS}^L}{\epsilon^K} \\
&= \frac{\partial \ln\left(\frac{P^L L}{P^M M}\right)}{\partial \ln K} \times \frac{\left(1-\mathrm{VCS}^L\right)\mathrm{VCS}^L}{\epsilon^K}, \qquad \text{(B.5)}
\end{aligned}
$$

where $\mathrm{VCS}^L = \frac{P^L L}{VC}$ is the labor share in short-run variable costs, which we have introduced in the third equality of both equations using Shephard's lemma in logs ($\frac{\partial \ln VC}{\partial \ln \widetilde{P}^L} = \frac{\partial VC}{\partial \widetilde{P}^L}\frac{\widetilde{P}^L}{VC} = \frac{\widetilde{P}^L \widetilde{L}}{VC} = \frac{P^L L}{VC}$). Expressions in the last equalities in (B.4)–(B.5) are obtained via substitution using the following output/capital elasticities of the relative demand for labor:

$$
\begin{aligned}
\frac{\partial \ln\left(\frac{P^L L}{P^M M}\right)}{\partial \ln Y^*} &= \left(\frac{1}{1-\mathrm{VCS}^L}\right)\frac{\partial \mathrm{VCS}^L}{\partial \ln Y^*},\\
\frac{\partial \ln\left(\frac{P^L L}{P^M M}\right)}{\partial \ln K} &= \left(\frac{1}{1-\mathrm{VCS}^L}\right)\frac{\partial \mathrm{VCS}^L}{\partial \ln K},
\end{aligned}
$$

both of which are easy to verify after recognizing that, at the optimum, $\frac{P^L L}{P^M M} = \frac{\mathrm{VCS}^L}{1-\mathrm{VCS}^L}$.

From (B.4)–(B.5), it is evident that forcing the firm's short-run expansion path in (L, M) space to be linear—via homothetic separability restriction, per (B.3)—is what "turns off" the endogenous response of RTS to changes in factor prices and, in particular, $\widetilde{P}^L$ which may be induced by labor-augmenting innovations in φ.

B.3 Implications for price path-through and shutdown rule

FACTOR-TO-OUTPUT PRICE PASS-THROUGH.—On the supply side, scale invariance of relative variable input allocations rules out what Lashkari et al. (2024) call the "returns to scale channel" in the transmission of factor price or factor-specific productivity shocks onto the firm's output price.

Intuitively, the firm maximizes ex-ante short-run profits by equating the revenue and variable costs at the margin: $\mathrm{MR} = \epsilon^Y \times \frac{VC}{Y^*}$, where the right-hand side equals $\frac{\partial \ln VC}{\partial \ln Y^*}\frac{VC}{Y^*} = \frac{\partial VC}{\partial Y^*}$ which is the ex-ante marginal cost MC. Assuming constant markups, which implies proportionality between the output price P^Y and the marginal revenue MR, it is not difficult to show that the pass-through of a change in the effective price of labor (say, due to a positive labor-augmenting technological innovation), in general, is

$$\frac{\partial \ln P^Y}{\partial \ln \widetilde{P}^L} = \left(1+\epsilon^D\right)^{-1}\left[\frac{\partial \ln \epsilon^Y}{\partial \ln \widetilde{P}^L} + \frac{\partial \ln VC}{\partial \ln \widetilde{P}^L}\right] \propto \mathrm{VCS}^L\left(\frac{1-\mathrm{VCS}^L}{\epsilon^Y} \times \frac{\partial \ln\left(\frac{P^L L}{P^M M}\right)}{\partial \ln Y^*} + 1\right),$$

where $\epsilon^D \equiv \frac{\partial \ln Y^*}{\partial \ln P^Y}$ is a constant price elasticity of demand, and we have used the results from Appendix B.2.

If permitted, scale effects on the variable input mix—through $\frac{\partial}{\partial \ln Y^*}\ln\left(\frac{P^L L}{P^M M}\right)$—allow for the possibility that firms of different size may enjoy differential income benefits from labor-saving technical change. Scale invariance of homothetic short-run production functions, however, imposes that the relative demand for labor does not respond to scale change in the short run, viz., $\frac{\partial}{\partial \ln Y^*}\ln\left(\frac{P^L L}{P^M M}\right) = 0$, leaving only the labor intensity of costs reflected in VCS^L to influence the pass-through.

SHUTDOWN RULE.—Homothetic separability between variable and fixed inputs also has non-trivial implications for the firm's shutdown considerations in the short run.

Concretely, recall the textbook shutdown condition, which is for the firm to cease its operations in the short run, incurring losses equal to the fixed cost, if the average variable cost exceeds the output price, or equivalently, if

$$B^{\text{shut}\downarrow} \equiv \frac{AVC}{P^Y} > 1, \tag{B.6}$$

where $AVC = \frac{VC}{Y^*}$ is the ex-ante average variable cost. At the firm's optimum, $P^Y(1+\frac{1}{\epsilon^D}) = \epsilon^Y \times AVC$, where ϵ^D is constant under the constant-price-elasticity demand structure. Substituting, we thus get that the relevant AVC-to-P^Y ratio equals the ratio of a constant markup and the output scale elasticity of the variable cost: $B^{\text{shut}\downarrow} = (1+\frac{1}{\epsilon^D})/\epsilon^Y$. This makes it clear that the orthogonality of ϵ^Y with respect to productivity-adjusted factor prices implied by homothetic separability, among other things, also imposes that factor price shocks have no influence the shutdown condition. This is a direct consequence of scale invariance of relative variable input allocations, exemplifying how homothetic separability may constrict the firm's behavior.

Analogous to the discussion of price pass-through, with markups assumed constant in order to focus on supply-side effects, we log-differentiate $B^{\text{shut}\downarrow}$ with respect to $\widetilde{P}^L$ to arrive at

$$\frac{\partial \ln B^{\text{shut}\downarrow}}{\partial \ln \widetilde{P}^L} = -\frac{\partial \ln\left(\epsilon^Y\right)}{\partial \ln \widetilde{P}^L} = -\frac{\partial \ln\left(\frac{P^L L}{P^M M}\right)}{\partial \ln Y^*} \times \frac{\left(1-\mathrm{VCS}^L\right)\mathrm{VCS}^L}{\epsilon^Y}, \tag{B.7}$$

where we have used the results from (B.4) in the second equality.

Under homothetic separability which imposes $\frac{\partial}{\partial \ln Y^*}\ln\left(\frac{P^L L}{P^M M}\right) = 0$ owing to scale invariance of the relative variable input demand, the shutdown elasticity in (B.7) is zero, whereby shocks to the effective wage have no influence on the firm's shutdown rule. As a consequence, $B^{\text{shut}\downarrow}$ is structurally pinned by the firm's scale of operation and its capital stock, with no direct dependence on variable factor prices or labor-augmenting productivity φ. Two firms sharing the same $(K, Y^*)'$ but facing different effective wages in the current period must confront identical shutdown conditions. Generally, however, this need not be the case.

From (B.7), we see that the elasticity $\frac{\partial \ln B^{\text{shut}\downarrow}}{\partial \ln \widetilde{P}^L}$ is generally non-zero, depends on the labor intensity of costs, scale elasticity of costs, as well as the shape of the short-run expansion path as captured by $\frac{\partial}{\partial \ln Y^*} \ln\left(\frac{P^L L}{P^M M}\right)$. In fact, the latter controls the sign of $\frac{\partial \ln B^{\text{shut}\downarrow}}{\partial \ln \widetilde{P}^L}$. Recall that, under constant markups, $P^Y \propto MC$ and $B^{\text{shut}\downarrow} \propto \frac{AVC}{MC} = \frac{1}{\epsilon^Y}$. Then, whether an increase in the effective wage makes shutdown in the short run more or less likely reduced to whether a rise in $\widetilde{P}^L$ increases MC proportionally more or less than AVC. When $\frac{\partial}{\partial \ln Y^*} \ln\left(\frac{P^L L}{P^M M}\right) > 0$ implying that the expansion path tilts toward labor at larger scale (i.e., labor's share in variable costs rises with output), production is more labor-intensive at the margin than on average. An effective wage increase then raises MC more than AVC, raising ϵ^Y and lowering $B^{\text{shut}\downarrow}$. Because the output price is pinned to MC via the markup, P^Y absorbs more of the shock than AVC does, making shutdown less likely at fixed Y^*. Conversely, when in $\frac{\partial}{\partial \ln Y^*} \ln\left(\frac{P^L L}{P^M M}\right) < 0$ the short-run expansion path tilts toward materials, MC is less labor-exposed than AVC, and thus an effective wage increase worsens the shutdown condition by pushing $B^{\text{shut}\downarrow}$ up and making shutdown more likely. In either case, the scale-dependent composition of variable costs creates a wedge between the marginal and average cost responses to effective factor price shocks that is absent under homothetic separability.

C Data Description

We use publicly available firm/plant-level panel datasets for manufacturing production from three countries: China (1998–2007), Colombia (1981–1991), and Chile (1979–1996). These data have been widely documented and used in the literature; e.g., see Brandt et al. (2012), Brandt et al. (2017), Malikov and Zhao (2023), Levinsohn and Petrin (2003), Grieco et al. (2016), Gandhi et al. (2020), Demirer (2025), Raval (2023a), and Raval (2023b). This appendix briefly describes the data sources and the processing methods for each country.

China.—The data on the Chinese manufacturing firms are taken from Chinese Industrial Enterprises Database. The database is conducted by China's National Bureau of Statistics (NBS) with a purpose to compute GDP statistics. The construction of production variables follows previous studies that have used the same database, e.g., Brandt et al. (2012), Zhao et al. (2020), Malikov et al. (2024), and Malikov and Zhao (2023). Specifically, firms' revenue is defined as the gross industrial output value, and to get its deflated value, we use an industry-level producer price index. Labor is measured as the number of employees. Materials are the total expense on intermediate inputs, including raw materials and other production-related inputs, deflated by the purchasing price index for industrial inputs. Firms' capital stock is calculated using the perpetual inventory method following Brandt et al. (2012). Price indices are obtained from the NBS and the World Bank. The financial variables are measured in thousands of real RMB.

To remove possibly misrecorded observations and outliers, we (i) drop the top and the bottom 1st percentiles of production variables and the labor-to-material ratio, including negative and missing values, (ii) drop firms with number of employees fewer than 8, since the database is supposed to record only “above-scale" firms, and (iii) exclude observations with the foreign equity share, labor-to-revenue or material-to-revenue shares falling outside the unit interval. Given that state firms have different objectives compared with private firms in China, we also remove state-owned firms from our analysis. Lastly, following Lee et al. (2023), we also exclude firms that switch their ownership status back and forth more than once in our sample. Our sample period runs from 1998 to 2007, and the operational sample is an unbalanced panel of 452,955 firms with a total of 1,415,697 observations.

Colombia and Chile.—Both the Colombian and Chilean manufacturing plant data are taken from the replication package of Raval (2023b). The Colombian dataset is originally from the annual Colombian Manufacturing census provided by the Departamento Administrativo Nacional de Estadistica. It covers years between 1981 and 1991, and contains about 7,000 plants per year. Plants with less than 10 employees are excluded in 1983 and 1984. The Chilean dataset is from the Chilean annual census of the manufacturing sector, Encuesta Nacional Industrial Anual (ENIA). It spans the years 1979 to 1996, covering all Chilean manufacturing plants with at least 10 employees and containing about 5,000 plants per year.

The detailed variable construction is variable in the appendix of Raval (2023b). Here, we provide a brief summary. For both datasets, a capital stock is constructed for each type of capital available using a perpetual inventory method, and the capital input is the sum of the stock of each type multiplied by its rental rate plus rental payments. Labor is measured as the total number of workers. The material input includes both energy costs and raw materials, which are deflated at the 3 digit ISIC level. Revenue is total sales, calculated as total production value (both domestic sales and exports, and sales to other establishments of the same company), plus the difference between the end year and beginning year value of inventories of finished goods. Real sales are nominal sales deflated by the output deflator at the 3 digit ISIC level.

To remove possibly misrecorded observations and outliers, we (i) drop the top and the bottom 1st percentiles of production variables and the labor-to-material ratio, including negative and missing values and (ii) exclude observations with the labor-to-revenue or material-to-revenue shares falling outside the unit interval.

SUPPLEMENTARY ONLINE APPENDIX

O1 Invariance of Elasticity of Substitution to Input Rescaling

Consider the production function $F(K,\widetilde{L},M)$, where the rescaled input $\widetilde{L} = e^{\varphi}L$ is "effective labor." In what follows, we show both Allen-Uzawa and Morishima ES are invariant to this input rescaling.

Recall that Allen-Uzawa and Morishima elasticities are defined as [see eqs. (2.4)–(2.5)]

$$\mathrm{AES}_{pq} = \frac{\sum_s X_s F_s}{X_p X_q}\frac{\mathrm{C}_{pq}}{|\overline{\mathrm{B}}|}, \qquad \mathrm{MES}_{pq} = \frac{F_p}{X_q}\frac{\mathrm{C}_{pq}}{|\overline{\mathrm{B}}|} - \frac{F_p}{X_p}\frac{\mathrm{C}_{pp}}{|\overline{\mathrm{B}}|}. \tag{O1.1}$$

Using the notation G_X, G_Z and G_{XZ} to denote first- and second-order partials of some function G with respect to its arguments, we have that the production-function derivatives transform as $F_L = e^{\varphi}F_{\widetilde{L}}, F_{LL} = e^{2\varphi}F_{\widetilde{L}\widetilde{L}}, F_{KL} = e^{\varphi}F_{K\widetilde{L}}, F_{ML} = e^{\varphi}F_{M\widetilde{L}}$ because $\widetilde{L} = e^{\varphi}L$. Therefore, $\sum_s X_s F_s = KF_K + LF_L + MF_M = KF_K + \widetilde{L}F_{\widetilde{L}} + MF_M$, so the revenue term is invariant. Further, since the bordered Hessian for $F(K,e^{\varphi}L,M)$ contains one row and one column corresponding to the first and second derivatives of L, i.e.,

$$\overline{\mathrm{B}} = \begin{pmatrix} 0 & F_K & F_L & F_M \\ F_K & F_{KK} & F_{KL} & F_{KM} \\ F_L & F_{LK} & F_{LL} & F_{LM} \\ F_M & F_{MK} & F_{ML} & F_{MM} \end{pmatrix} = \begin{pmatrix} 0 & F_K & e^{\varphi}F_{\widetilde{L}} & F_M \\ F_K & F_{KK} & e^{\varphi}F_{K\widetilde{L}} & F_{KM} \\ e^{\varphi}F_{\widetilde{L}} & e^{\varphi}F_{K\widetilde{L}} & e^{2\varphi}F_{\widetilde{L}\widetilde{L}} & e^{\varphi}F_{M\widetilde{L}} \\ F_M & F_{MK} & e^{\varphi}F_{M\widetilde{L}} & F_{MM} \end{pmatrix},$$

its determinant and cofactors scale as

$$|\overline{\mathrm{B}}| = e^{2\varphi}|\widetilde{\overline{\mathrm{B}}}|,\ \mathrm{C}_{KL} = e^{\varphi}\widetilde{\mathrm{C}}_{K\widetilde{L}},\ \mathrm{C}_{LL} = \widetilde{\mathrm{C}}_{\widetilde{L}\widetilde{L}},\ \mathrm{C}_{KK} = e^{2\varphi}\widetilde{\mathrm{C}}_{KK},\ \mathrm{C}_{KM} = e^{2\varphi}\widetilde{\mathrm{C}}_{KM},\ \mathrm{C}_{MM} = e^{2\varphi}\widetilde{\mathrm{C}}_{MM},\ \mathrm{C}_{LM} = e^{\varphi}\widetilde{\mathrm{C}}_{\widetilde{L}M},$$

where $\widetilde{\overline{\mathrm{B}}}$ is the bordered Hessian formed from $F(K,\widetilde{L},M)$ as if $\widetilde{L}$ were an unscaled input:

$$\widetilde{\overline{\mathrm{B}}} = \begin{pmatrix} 0 & F_K & F_{\widetilde{L}} & F_M \\ F_K & F_{KK} & F_{K\widetilde{L}} & F_{KM} \\ F_{\widetilde{L}} & F_{K\widetilde{L}} & F_{\widetilde{L}\widetilde{L}} & F_{M\widetilde{L}} \\ F_M & F_{MK} & F_{M\widetilde{L}} & F_{MM} \end{pmatrix},$$

with $\widetilde{\mathrm{C}}_{pq}$ denoting the corresponding cofactors.

Consider MES_{LK}. Using (O1.1) and substituting the scaling relationships yields

$$\mathrm{MES}_{LK} = \frac{F_L}{K}\frac{\mathrm{C}_{LK}}{|\overline{\mathrm{B}}|} - \frac{F_L}{L}\frac{\mathrm{C}_{LL}}{|\overline{\mathrm{B}}|} = \frac{e^{\varphi}F_{\widetilde{L}}}{K}\frac{e^{\varphi}\widetilde{\mathrm{C}}_{K\widetilde{L}}}{e^{2\varphi}|\widetilde{\overline{\mathrm{B}}}|} - \frac{e^{\varphi}F_{\widetilde{L}}}{e^{-\varphi}\widetilde{L}}\frac{\widetilde{\mathrm{C}}_{\widetilde{L}\widetilde{L}}}{e^{2\varphi}|\widetilde{\overline{\mathrm{B}}}|} = \frac{F_{\widetilde{L}}}{K}\frac{\widetilde{\mathrm{C}}_{K\widetilde{L}}}{|\widetilde{\overline{\mathrm{B}}}|} - \frac{F_{\widetilde{L}}}{\widetilde{L}}\frac{\widetilde{\mathrm{C}}_{\widetilde{L}\widetilde{L}}}{|\widetilde{\overline{\mathrm{B}}}|} = \mathrm{MES}_{\widetilde{L}K}.$$

Similarly, we can show the equivalence of other Morishima ES.

For AES_{LK}, we have

$$\text{AES}_{LK} = \frac{\sum_s X_s F_s}{LK}\frac{\text{C}_{KL}}{|\overline{\text{B}}|} = \frac{\sum_s \widetilde{X}_s F_s}{e^{-\varphi}\widetilde{L}K}\frac{e^{\varphi}\widetilde{\text{C}}_{K\widetilde{L}}}{e^{2\varphi}|\widetilde{\overline{\text{B}}}|} = \frac{\sum_s \widetilde{X}_s F_s}{\widetilde{L}K}\frac{\widetilde{\text{C}}_{K\widetilde{L}}}{|\widetilde{\overline{\text{B}}}|} = \text{AES}_{\widetilde{L}K}.$$

The equivalence of other Allen-Uzawa ES can be shown in a similar way.

O2 Identification of Other Parametric Production Functions

In this appendix, we consider nonneutral production technologies that are not homothetically separable, taking parametric forms other than the translog studied in Section 3. Below we sketch out the steps in structural model identification for two alternative functional forms of $F(\cdot)$: the Generalized Cobb-Douglas (GCD) that is nonseparable in variable-vs-fixed inputs, and the Generalized Leontief (GL) that is neither homothetic nor separable. We do so under the same model assumptions as those stipulated in Section 3 except in Assumption 1, instead of assuming that $F(\cdot)$ takes the translog form in (3.1), in what follows we let it be of the form in (O2.1) or (O2.8).

O2.1 Generalized Cobb-Douglas

Consider the GCD nonneutral production function (Fuss et al., 1978; Griffin et al., 1987):

$$\ln F(K,\widetilde{L},M) = \beta_k \ln K + \beta_l \ln\widetilde{L} + \beta_m \ln M + \beta_{kl}\ln\left(\tfrac{1}{2}(K+\widetilde{L})\right) + \beta_{km}\ln\left(\tfrac{1}{2}(K+M)\right) + \beta_{lm}\ln\left(\tfrac{1}{2}(\widetilde{L}+M)\right). \tag{O2.1}$$

Note that it is homothetic in all its inputs but $F(\cdot)$ is not separable in the partition $(K,(\widetilde{L},M))$ unless $\beta_{kl} = \beta_{km} = 0$. The production function is homogeneous of degree $(\beta_k + \beta_l + \beta_m + \beta_{kl} + \beta_{km} + \beta_{lm})$.

To begin with, it is not difficult to show that the first stage of our identification methodology in Section 3.1 remains valid. Intuitively, this stage involves a reduced-form revenue production function of an unspecified form, no matter the true parametric form of $F(\cdot)$. As such, we continue to be able to isolate a composite noise term η^* via nonparametric projection of $\widetilde{r}$ on $\left(\ln(K),\ln(L),\ln(M),\frac{P^L L}{P^M M},\chi'\right)'$ and time effects, also identifying Λ^*, S^{L*} and S^{M*} by extension.

For the second stage, under the GCD specification, the firm's static profit-maximizing conditions, when expressed in the share form, become

$$S^L = \left(\beta_l^* + \frac{\widetilde{L}}{\widetilde{L}+K}\beta_{kl}^* + \frac{\widetilde{L}}{\widetilde{L}+M}\beta_{lm}^*\right)\Lambda^* e^{-\eta^*}, \tag{O2.2}$$

$$S^M = \left(\beta_m^* + \frac{M}{\widetilde{L}+M}\beta_{lm}^* + \frac{M}{K+M}\beta_{km}^*\right)\Lambda^* e^{-\eta^*}, \tag{O2.3}$$

where all variable and parameter definitions (including the asterisk notation) remain exactly as in the main text. These two equations are respective counterparts of (3.16) and (3.17).

Because the share equations continue to depend only on labor-augmenting productivity (through $\widetilde{L} = e^{\varphi}L$), we can invert them to construct proxies for φ. Solving for φ from the labor share equation (O2.2), we obtain

$$\varphi = \ln\left(\frac{S^{L*} - \beta_l^*}{\beta_l^* + \beta_{lm}^* - S^{L*}}\right) - \ln\left(\tfrac{L}{M}\right), \tag{O2.4}$$

where we have imposed $\beta_{kl} = 0$ for mathematical convenience to avoid square-root terms and get a simpler analytical expression. We would then use this proxy to substitute for φ in the Markov process for factor-augmenting technical change ($\varphi = \alpha_1\mathbb{L}[\varphi] + \alpha_2\mathbb{L}[Z_1] + \epsilon$) to obtain a counterpart expression to (3.18) [omitted], the corresponding moment conditions for which (à la (3.19)) would be

$$\mathbb{E}\Big[\epsilon \mid \mathbb{L}\big[S^{L*}\big], \mathbb{L}[l-m], \mathbb{L}[Z_1]\Big] = 0 \quad \text{and} \quad \mathbb{E}\big[\epsilon\big] = 0,$$

from which, besides secondary parameters, we can identify the unknown production-function parameters of interest β_l^* and β_{lm}^*.

Similarly, we can invert the material input share equation (O2.3) to solve for φ:

$$\varphi = \ln\left(\frac{\beta_{lm}^*(K+M)}{K\left(S^{M*} - \beta_m^*\right) - M\left(\beta_{km}^* + \beta_m^* - S^{M*}\right)} - 1\right) - \ln\left(\tfrac{L}{M}\right), \tag{O2.5}$$

where β_{lm}^* is already identified/known, but β_m^* and β_{km}^* remain unknown. To identify these two parameters, we could use the φ law of motion again to obtain the counterpart of (3.20) [omitted], with the corresponding moment conditions (3.21) becoming

$$\mathbb{E}\Big[\epsilon\big(1, \mathbb{L}\big[S^{M*}\big], K, \mathbb{L}[M]\big)'\Big] = 0.$$

Finally, defining $\widetilde{r}^* = \widetilde{r} - \beta_l^* \ln\widetilde{L} - \beta_m^* \ln M - \beta_{km}^* \ln\left(\frac{1}{2}(K+M)\right) - \beta_{lm}^* \ln\left(\frac{1}{2}(\widetilde{L}+M)\right) - \eta^*$, which includes all elements of the production function already identified in the previous steps, we can identify the remaining production-function parameters by writing ω as (analogous to that in (3.22))

$$\omega = \frac{\sigma}{1+\sigma}\widetilde{r}^* - \beta_k k + \frac{1}{1+\sigma}\overline{y} - \frac{\sigma}{1+\sigma}\rho'\chi \tag{O2.6}$$

in order to substitute this proxy into the Markov process for Hicksian productivity ($\omega = \gamma_0 + \gamma_1\mathbb{L}[\omega] + \gamma_2\mathbb{L}[Z_2] + \varepsilon$), arriving at the third-stage equation akin to that in (3.23) with the corresponding moment restrictions (3.24) simplifying to

$$\mathbb{E}\Big[\varepsilon^* \Big| \mathbb{L}\big[\widetilde{r}^*\big], k, \mathbb{L}[k], \mathbb{L}[\overline{y}], \mathbb{L}[\chi], \mathbb{L}[Z_2]\Big] = 0 \quad \text{and} \quad \mathbb{E}\big[\varepsilon^*\big] = 0. \tag{O2.7}$$

O2.2 Generalized Leontief

Now consider the GL case. Adopting a generic formulation $F(X)=\sum_i \beta_i X_i^{1/2}+\sum_i\sum_j \beta_{ij}X_i^{1/2}X_j^{1/2}$ to our setting (Fuss et al., 1978), we have the following nonneutral production function:

$$\begin{aligned} F(K,\widetilde{L},M)=&\beta_k K^{1/2}+\beta_l\widetilde{L}^{1/2}+\beta_m M^{1/2}+\beta_{kk}K+\beta_{ll}\widetilde{L}+\beta_{mm}M+\\ &\beta_{kl}K^{1/2}\widetilde{L}^{1/2}+\beta_{km}K^{1/2}M^{1/2}+\beta_{lm}M^{1/2}\widetilde{L}^{1/2}, \end{aligned} \tag{O2.8}$$

which is stipulated in levels and not logs. Note that, unless $\beta_k=\beta_l=\beta_m=0$, the function (O2.8) is not homothetic in inputs and, unless $\beta_{kl}=\beta_{km}=0$, it is also not separable in the partition $(K,(\widetilde{L},M))$.

To identify the above GL production function, we follow the same three stages. The first stage is unchanged. For the second stage, the variable input share equations become

$$S^L=\left(\tfrac{1}{2}\beta^*_{kl}K^{1/2}\widetilde{L}^{-1/2}+\tfrac{1}{2}\beta^*_l\widetilde{L}^{-1/2}+\tfrac{1}{2}\beta^*_{lm}M^{1/2}\widetilde{L}^{-1/2}+\beta^*_{ll}\right)\frac{\widetilde{L}}{F(K,\widetilde{L},M)}\Lambda^* e^{-\eta^*}, \tag{O2.9}$$

$$S^M=\left(\tfrac{1}{2}\beta^*_{lm}M^{-1/2}\widetilde{L}^{1/2}+\tfrac{1}{2}\beta^*_{km}K^{1/2}M^{-1/2}+\tfrac{1}{2}\beta^*_m M^{-1/2}+\beta^*_{mm}\right)\frac{M}{F(K,\widetilde{L},M)}\Lambda^* e^{-\eta^*}, \tag{O2.10}$$

using which we can derive proxies for labor-augmenting productivity φ. To get a relatively simpler expression, for mathematical convenience we impose that $\beta_{ll}=0$. Then, from (O2.9) we can solve for $\widetilde{L}^{1/2}$ as

$$\widetilde{L}^{1/2}=2\frac{-\beta_k K^{1/2}S^{L*}-\beta_{kk}KS^{L*}-\beta_{km}K^{1/2}M^{1/2}S^{L*}-\beta_m M^{1/2}S^{L*}-\beta_{mm}MS^{L*}}{\left(2S^{L*}-c\right)\left(\beta_{kl}K^{1/2}+\beta_{lm}M^{1/2}+\beta_l\right)},$$

where $c=1+\frac{1}{\sigma}$. For ease of notation, label the right-hand side of the equation as some function $g_L(K,L,M,S^{L*};\boldsymbol{\beta},\sigma)$ where $\boldsymbol{\beta}$ denotes all unknown β parameters. According to the definition of $\widetilde{L}$ and taking the logarithms of the above proxy equation, we have

$$\varphi=-\ln L+2\ln g_L(K,L,M,S^{L*};\boldsymbol{\beta},\sigma). \tag{O2.11}$$

Similarly, we can solve $\widetilde{L}^{1/2}$ from the materials share equation (O2.10) to get

$$\widetilde{L}^{1/2}=\frac{-2\beta_k K^{1/2}S^{M*}-2\beta_{kk}KS^{M*}+\beta_{km}K^{1/2}M^{1/2}\left(c-2S^{M*}\right)+\beta_m M^{1/2}\left(c-2S^{M*}\right)-2\beta_{mm}M\left(S^{M*}-c\right)}{2S^{M*}\left(\beta_{kl}K^{1/2}+\beta_l\right)+\beta_{lm}M^{1/2}\left(2S^{M*}-c\right)}.$$

Denoting the right-hand side of the equation as some other function $g_M(K,L,M,S^{M*};\boldsymbol{\beta},\sigma)$ and logging the above equation, we get another proxy:

$$\varphi=-\ln L+2\ln g_M(K,L,M,S^{M*};\boldsymbol{\beta},\sigma). \tag{O2.12}$$

Note that, unlike in the translog or the GCD case, both φ proxies under the GL specification admit *all* production-function coefficients. As such, the second stage of our identification methodology would readily deliver all $\boldsymbol{\beta}$. To operationalize this stage, we could use the proxy in (O2.11) and/or

(O2.12) to substitute for φ in its Markov process, forming identifying moment conditions that exploit mean orthogonality between the productivity innovation ϵ and variables in $\mathbb{L}[\mathscr{I}]$. However, a parameter normalization would be in order as both $g_L(K,L,M,S^{L*};\boldsymbol{\beta},\sigma)$ and $g_M(K,L,M,S^{M*};\boldsymbol{\beta},\sigma)$ effectively are functions of the ratios of "β" coefficients. Hence, $\boldsymbol{\beta}$ is only identified up to scale in this case. A natural choice for normalization here would be to set the coefficient on $\widetilde{L}^{1/2}$ equal to 1 ($\beta_l = 1$), whereby β_l and φ are not separably identifiable as is standard in a CES specification (e.g., see Doraszelski and Jaumandreu, 2018).

However, note that σ now appears both in the second stage above and in our third stage. We suggest a joint estimation of these two steps. Specifically, redefine $\widetilde{r}^* \equiv \widetilde{r} - \ln F(K,\widetilde{L},M;\boldsymbol{\beta}) - \eta^*$ to now write ω as $\omega = \frac{\sigma}{1+\sigma}\widetilde{r}^* + \frac{1}{1+\sigma}\overline{y} - \frac{\sigma}{1+\sigma}\rho'\chi$ and implement the above stage two and our stage three together, where the former is based on the φ process and the latter is based on the ω process. This joint implementation identifies all unknown parameters $\left(\boldsymbol{\beta},\sigma,\alpha_1,\alpha_2,\rho\right)'$.

O3 Identification of the Translog Production Function under Perfect Competition: A Special Case

The identification scheme for nonneutral production functions of the translog form outlined in Section 3.1 assumes a monopolistically competitive output market. In this appendix, we consider its simplification when, following canonical proxy variable estimators, we assume that firms have no monopoly power. To this end, consider the revised Assumption 3.

Assumption 3$'$ *Firms in a given industry/segment are price-takers in the output market, facing a homogeneous price* $P^Y = \overline{P}$.

In such a case, we essentially have that $\sigma = -\infty$, as well as $\rho = 0$ and $\xi = 0$ in the inverse demand equation (3.9). This simplifies all key estimation equations, allowing us to shed the asterisk off all β^* parameters (since $(1+\frac{1}{\sigma}) = 1$) as well as drop demand shifters ξ and $\overline{y}$ from the revenue production function. In fact, under Assumption 3$'$, there is no longer a distinction between the revenue and quantity production functions because the deflated revenue equals the output quantity: $\frac{P^Y Y}{\overline{P}} = Y$. The translog production function in (3.13) simplifies to

$$y = \beta_k k + \beta_l[\varphi + l] + \beta_m m + \beta_{kl} k[\varphi + l] + \beta_{km} km + \beta_{lm}[\varphi + l]m + \tfrac{1}{2}\beta_{kk}k^2 + \tfrac{1}{2}\beta_{ll}[\varphi + l]^2 + \tfrac{1}{2}\beta_{mm}m^2 + \omega + \eta. \tag{O3.1}$$

Following the same arguments as those fleshed out in Section 3.1, we obtain simplified analogues of all key estimating equations. Namely, the "reduced form" of (O3.1) for stage one becomes

$$y = \Phi\left(k,l,m,\frac{P^L L}{P^M M}\right) + \eta, \tag{O3.2}$$

whereas share equations for stage two simplify to

$$S^L = \left(\beta_l + \beta_{kl}k + \beta_{lm}m + \beta_{ll}l + \beta_{ll}\varphi\right)\Lambda e^{-\eta}, \tag{O3.3}$$

$$S^M = \left(\beta_m + \beta_{km}k + \beta_{mm}m + \beta_{lm}l + \beta_{lm}\varphi\right)\Lambda e^{-\eta}. \tag{O3.4}$$

The identification procedure gets streamlined as follows.

Stage One.—Isolate shock η (and by extension, $\Lambda = \mathbb{E}(e^{\eta})$) from (O3.2) based on the moments

$$\mathbb{E}\Big[\eta \Big|\, k, l, m, \tfrac{P^L L}{P^M M}\Big] = 0 \quad \text{and} \quad \mathbb{E}\big[\eta\big] = 0. \tag{O3.5}$$

Stage Two.—Solving for φ from each one of optimal share equations in (O3.3) and (O3.4):

$$\varphi = \frac{1}{\beta_{ll}}S^{L*} - \frac{\beta_l}{\beta_{ll}} - \frac{\beta_{kl}}{\beta_{ll}}k - \frac{\beta_{lm}}{\beta_{ll}}m - l \quad \text{and} \quad \varphi = \frac{1}{\beta_{lm}}S^{M*} - \frac{\beta_m}{\beta_{lm}} - \frac{\beta_{km}}{\beta_{lm}}k - \frac{\beta_{mm}}{\beta_{lm}}m - l,$$

where we slightly redefine $S^{L*} \equiv S^L e^{\eta}/\Lambda$ and $S^{M*} \equiv S^M e^{\eta}/\Lambda$, and substituting these proxies into the φ law of motion (3.7), we obtain analogues of second-stage equations (3.18) and (3.20):

$$\begin{aligned} S^{L*} = {} & \beta_l(1-\alpha_1) + \alpha_1\mathbb{L}\left[S^{L*}\right] + \\ & \beta_{kl}\left(k - \alpha_1\mathbb{L}[k]\right) + \beta_{ll}\left(l - \alpha_1\mathbb{L}[l]\right) + \beta_{lm}\left(m - \alpha_1\mathbb{L}[m]\right) + \alpha_2\beta_{ll}\mathbb{L}[Z_1] + \epsilon^*, \end{aligned} \tag{O3.6}$$

which identifies $(\alpha_1, \alpha_2, \beta_l, \beta_{ll}, \beta_{kl}, \beta_{lm})'$ on the basis of moments in (3.19), and

$$\widetilde{S}^{M*} = \beta_m(1-\alpha_1) + \beta_{km}\triangle k + \beta_{mm}\triangle m + \epsilon^{\dagger}, \tag{O3.7}$$

which identifies β_m, β_{km}, and β_{mm} based on the moments in (3.21).

Stage Three.—Collecting all already identified parts of the production function as $y^* \equiv y - \beta_l\widetilde{l} - \beta_m m - \beta_{kl}k\widetilde{l} - \beta_{km}km - \beta_{lm}\widetilde{l}m - \frac{1}{2}\beta_{ll}\widetilde{l}^2 - \frac{1}{2}\beta_{mm}m^2 - \eta$ and making use of the ω law of motion in (3.8), we arrive at the analogue of (3.23):

$$y^* = \gamma_0 + \gamma_1\mathbb{L}[y^*] + \beta_k\left(k - \gamma_1\mathbb{L}[k]\right) + \tfrac{1}{2}\beta_{kk}\left(k^2 - \gamma_1\mathbb{L}[k]^2\right) + \gamma_2\mathbb{L}[Z_2] + \varepsilon, \tag{O3.8}$$

from where we identify the remaining unknown parameters $(\gamma_0, \gamma_1, \gamma_2, \beta_k, \beta_{kk})'$ using

$$\mathbb{E}\Big[\varepsilon \Big|\, \mathbb{L}\left[y^*\right], k, \mathbb{L}[k], k^2, \mathbb{L}[k]^2, \mathbb{L}[Z_2]\Big] = 0 \quad \text{and} \quad \mathbb{E}\big[\varepsilon\big] = 0. \tag{O3.9}$$

O4 Additional Simulation Results

This appendix reports simulation designs in which the true production function deviates from the translog specification used in estimation. Across all designs, we keep fixed the panel structure

($N = 500, T = 10$), the demand, the ($\omega_{it}, \varphi_{it}$) productivity processes, and the capital accumulation equation described in Section 5. In each design, variable inputs (L, M) are generated by numerically solving the firm's static first-order conditions implied by the *true* production technology. In the estimation stage, however, we always fit the baseline second-order translog model, thereby intentionally introducing functional-form misspecification. Relative to the baseline translog DGP, we allow certain nuisance parameters (e.g., initial values of productivities and capital, as well as shock variances) to differ across designs. These adjustments are made to ensure that the simulated output and input choices are well-behaved, that monotonicity and curvature violations occur only rarely, and that the numerical solution of the firm's problem is stable.

We consider the following alternative production technologies: CES, Generalized Cobb-Douglas (GCD), and a higher-order generalized translog (GTL) that augments the translog with cubic terms. For each design, we report RTS and ES objects. Because they are observation-specific and can be sensitive to outliers, within each Monte-Carlo replication we first compute the median of each object across all firm-time observations. We then summarize performance across replications using the Monte-Carlo mean of these within-replication medians and the corresponding RMSE over 500 replications.

CES.—The true production function follows a CES form:

$$F(K,L,M) = \left(b_k K^{\rho} + e^{\varphi} L^{\rho} + b_m M^{\rho}\right)^{\nu/\rho}, \qquad \rho = \frac{\sigma - 1}{\sigma}. \tag{O4.1}$$

We set $b_k = b_m = 0.3$, the substitution elasticity to $\sigma = 1.5$, and consider three values of the scale parameter:

$$\nu \in \{0.9,\ 1.0,\ 1.1\}.$$

Table O.1 evaluates the performance of our translog estimator when the true production technology is (O4.1). Across all three scale DGPs, RTS are recovered very accurately. The simulated means remain very close to the true values with small RMSEs throughout, indicating that the translog approximation captures global scale properties of the CES technology well in finite samples.

For substitution elasticities, performance varies across input pairs. AES for (K, L) are estimated most precisely, while those for (K, M) exhibit moderate downward bias and larger dispersion. The largest discrepancies arise for the AES_{LM}, which is systematically overestimated and associated with relatively large RMSEs. In contrast, Morishima elasticities are generally estimated more accurately and display smaller dispersion across replications.

Overall, when the true technology is CES, the translog specification approximates scale effects and most Morishima substitution structure reliably, whereas Allen-Uzawa elasticities (particularly along the $(L, M)'$ margin) are more sensitive to functional-form misspecification.

Generalized Cobb-Douglas.—The true production function is generated following a GCD form

Table O.1: Performance under the CES DGP

		$\nu = 0.9$		$\nu = 1.0$		$\nu = 1.1$	
Object	True	Mean	RMSE	Mean	RMSE	Mean	RMSE
Returns to Scale							
RTS	ν	0.901	0.008	1.000	0.008	1.084	0.026
Allen-Uzawa Elasticity of Substitution							
KL	1.5	1.471	0.037	1.472	0.040	1.462	0.063
KM	1.5	1.357	0.150	1.364	0.143	1.364	0.146
LM	1.5	1.935	0.461	1.884	0.412	1.849	0.379
Morishima Elasticity of Substitution							
KL	1.5	1.466	0.041	1.467	0.044	1.457	0.066
LK	1.5	1.491	0.022	1.491	0.025	1.483	0.046
KM	1.5	1.385	0.120	1.394	0.112	1.395	0.117
MK	1.5	1.500	0.015	1.508	0.020	1.516	0.028
LM	1.5	1.617	0.124	1.615	0.123	1.614	0.122
ML	1.5	1.525	0.033	1.532	0.041	1.542	0.052

Table O.2: Performance under the GCD DGP

	$b_l = 0.20$			$b_l = 0.30$			$b_l = 0.40$		
Object	True	Mean	RMSE	True	Mean	RMSE	True	Mean	RMSE
Returns to Scale									
RTS	0.900	0.907	0.014	1.000	1.020	0.032	1.100	1.144	0.067
Allen-Uzawa Elasticity of Substitution									
KL	1.003	0.988	0.030	1.004	0.995	0.031	1.006	1.001	0.032
KM	1.003	0.980	0.031	1.003	0.984	0.029	1.005	0.986	0.030
LM	0.898	0.904	0.012	0.915	0.918	0.011	0.929	0.929	0.013
Morishima Elasticity of Substitution									
KL	1.003	0.984	0.030	1.004	0.990	0.029	1.005	0.995	0.030
LK	0.949	0.945	0.011	0.963	0.960	0.013	0.973	0.971	0.016
KM	1.003	0.982	0.030	1.004	0.987	0.029	1.005	0.991	0.029
MK	0.981	0.965	0.023	0.979	0.966	0.019	0.979	0.967	0.018
LM	0.927	0.927	0.006	0.937	0.938	0.006	0.947	0.947	0.006
ML	0.927	0.925	0.004	0.937	0.935	0.005	0.946	0.943	0.007

in logs:

$$\ln F(K,\widetilde{L},M) = b_k \ln K + b_l \log \widetilde{L} + b_m \ln M + b_{kl} \ln\Big(\tfrac{1}{2}(K+\widetilde{L})\Big) + b_{km} \ln\Big(\tfrac{1}{2}(K+M)\Big) + b_{lm} \ln\Big(\tfrac{1}{2}(\widetilde{L}+M)\Big), \tag{O4.2}$$

where we fix $b_k = 0.20$, $b_m = 0.50$, $b_{kl} = 0.025$, $b_{km} = 0.025$, $b_{lm} = -0.05$, and vary the labor coefficient:

$$b_l \in \{0.20,\ 0.30,\ 0.40\}.$$

This specification produces smooth deviations from Cobb-Douglas through log-averages of input pairs, offering a parsimonious form of nonseparability.

Table O.2 reports the performance of our translog estimator when the true technology follows the specification in (O4.2). Across all three values of b_l, the estimator recovers RTS reasonably well. For $b_l = 0.20$, the RTS estimate is nearly with no bias and small dispersion. As b_l increases, the estimated RTS exhibits a mild upward bias and larger RMSEs but, overall, scale elasticity remains well approximated.

Substitution elasticities are recovered with high accuracy. AES for all input pairs (K,L), (K,M) and (L,M) are estimated very close to their true values, with small RMSEs that are stable across designs. A similar pattern holds for MES: mean estimates are close to the true values for all input pairs, and RMSEs remain small as the labor coefficient increases.

Overall, when the true production technology is GCD, our translog estimator provides an excellent approximation. Both scaling and substitution patterns are recovered accurately, and misspecification effects remain quantitatively minor across all parameter configurations considered.

GENERALIZED TRANSLOG WITH CUBIC TERMS.—The GTL design augments the baseline second-order translog by adding cubic terms:

$$\ln F(K,\widetilde{L},M) = \ln F^{TL}(K,\widetilde{L},M) + b_{kkk}(\log K)^3 + b_{lll}(\log \widetilde{L})^3 + b_{mmm}(\log M)^3, \tag{O4.3}$$

where coefficients corresponding to the baseline quadratic translog $\ln F^{TL}(\cdot)$ are $b_k = 0.2$, $b_{kk} = -0.01$, $b_l = 0.25$, $b_{ll} = 0.05$, $b_m = 0.5$, $b_{mm} = 0.05$, $b_{kl} = 0.02$, $b_{km} = 0.02$, $b_{lm} = -0.05$. We set $b_{kkk} = 0.003$ and consider three cases:

$$(b_{lll}, b_{mmm}) \in \{(0,0),\ (0.003,0),\ (0.003,0.003)\}.$$

This DGP produces controlled departures from the second-order translog class: as additional cubic terms are introduced in variable inputs, the misspecification becomes more pronounced.

Table O.3 examines the performance of our second-order translog estimator when the true technology has cubic terms. When only the capital cubic term is present ($(b_{kkk}, b_{lll}, b_{mmm}) = (3,0,0)/10^3$), the estimator performs very well. The RTS estimator is essentially unbiased with small dispersion,

Table O.3: Performance under the GTL DGP

	$(b_{kkk}, b_{lll}, b_{mmm}) =$ $(3, 0, 0)/10^3$			$(b_{kkk}, b_{lll}, b_{mmm}) =$ $(3, 3, 0)/10^3$			$(b_{kkk}, b_{lll}, b_{mmm}) =$ $(3, 3, 3)/10^3$		
Object	True	Mean	RMSE	True	Mean	RMSE	True	Mean	RMSE
Returns to Scale									
RTS	1.282	1.284	0.009	1.286	1.301	0.131	1.303	1.295	0.060
Allen-Uzawa Elasticity of Substitution									
KL	0.993	0.987	0.059	1.022	0.279	0.772	0.991	0.872	0.585
KM	1.122	1.099	0.058	1.115	0.462	0.694	1.145	1.281	0.910
LM	1.446	1.425	0.034	1.459	1.560	0.187	1.510	1.313	0.223
Morishima Elasticity of Substitution									
KL	1.055	1.043	0.057	1.073	0.373	0.730	1.072	1.084	0.749
LK	1.213	1.206	0.021	1.239	0.955	0.292	1.243	1.125	0.224
KM	1.091	1.073	0.057	1.091	0.415	0.716	1.108	1.172	0.792
MK	1.198	1.177	0.041	1.200	0.767	0.463	1.226	1.295	0.615
LM	1.318	1.309	0.012	1.342	1.308	0.043	1.365	1.248	0.120
ML	1.354	1.338	0.019	1.357	1.343	0.037	1.402	1.336	0.082

and both Allen–Uzawa and Morishima elasticities are recovered accurately. Once an additional cubic term is introduced (in labor), approximation errors become substantial. RTS exhibit larger dispersion, and several substitution elasticities, particularly the $\text{AES}_{K,L}$ and $\text{AES}_{K,M}$, display severe bias and large RMSEs. Performance of Morishima elasticity estimators also deteriorates for these input pairs, although elasticities involving (L, M) remain relatively stable.

When cubic terms are introduced in all inputs, performance improves relative to the intermediate case in terms of the simulation means, but remains weaker than in the single-cubic design.[22] Whereas RTS are, again, estimated with moderate accuracy, substitution elasticities show large RMSEs. Overall, the GTL results demonstrate that the second-order translog specification is robust to mild higher-order curvature in a single factor but can become unstable when nonlinear departures are introduced simultaneously across multiple inputs.

O5 Additional Empirical Results

Table O.4 reports median estimates of the output elasticities of capital, labor, materials, their sum (i.e., RTS), as well as Allen-Uzawa and Morishima ES across five major industries in China, Colombia, and Chile. Besides the results from our not-homothetically-separable full-translog model, we also include estimates from the restricted model ("HS") that imposes homothetic separability on the translog production function via parameter constraints.

Output elasticities and the implied RTS are broadly similar across the two models in most

[22]One explanation for the worst performance of the intermediate case is that cubic curvature is introduced in some inputs but not others, generating asymmetric higher-order nonlinearity. This imbalance leads to distorted substitution patterns and large dispersion in the estimated elasticities.

industry-country cases. This indicates that the homothetic separability restriction has a limited effect on the recovery of these objects. In line with earlier studies, materials have the highest elasticity across all countries. The RTS are above one in China—implying increasing returns—while Colombia and Chile exhibit near-unitary returns to scale, with some exceptions like the Chilean apparel that shows high RTS.

The Allen-Uzawa and Morishima elasticities of substitution differ more substantially between the two models. Under homothetic separability, the theoretical restrictions on input substitution structure are visible in the HS columns: $\text{AES}_{KL} = \text{AES}_{KM}$, $\text{MES}_{KL} = \text{MES}_{KM}$, and $\text{MES}_{LM} = \text{MES}_{ML}$ hold by construction. Our unrestricted not-homothetically-separable model, by contrast, permits these elasticities to differ, and in many industries the discrepancies are economically meaningful. For instance, our model occasionally yields elasticities of opposite sign to those obtained under homothetic separability (see AES_{KL} in Colombia's wood industry and AES_{LM} in several Chilean industries), suggesting that the restriction can alter not only the magnitude but also the qualitative characterization of input substitutability.

Consider, also, the productivity results. We recover Hicks-neutral and labor-augmenting productivities from the estimated production function based on (3.4) and (3.22). To facilitate interpretation of growth over time, we normalize ω and φ to have a zero cross-firm mean in the first sample year and then exponentiate to convert log-productivity into level productivity. The resulting series $\exp(\omega)$ and $\exp(\varphi)$ are thus normalized on average in the first year so that the subsequent values may be interpreted as proportional changes in productivity relative to that baseline.

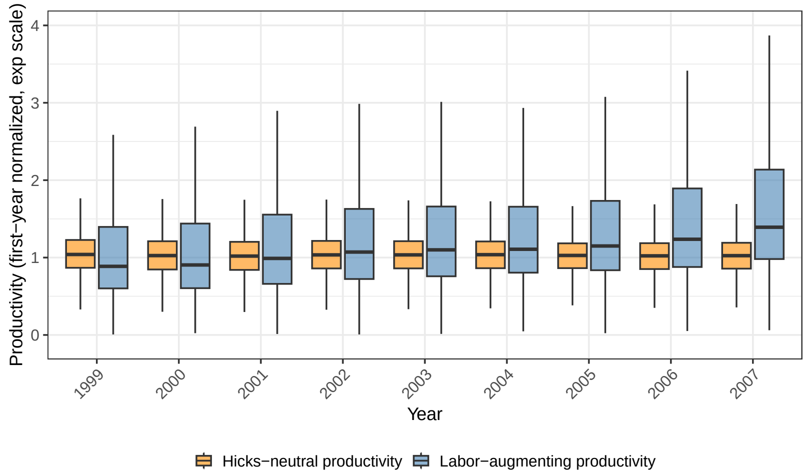
Figure O.1 plots the cross-firm distributions of $\exp(\omega)$ and $\exp(\varphi)$ by year as boxplots for China, Colombia, and Chile, with values above (below) one indicating productivity growth (decline) relative to the first-year baseline. Across the three panels, Hicks-neutral productivity remains relatively stable over time, with medians generally close to unity and only moderate dispersion. In contrast, labor-augmenting productivity exhibits larger cross-firm variation and clearer time trends. In China, the median of $\exp(\varphi)$ increases steadily over the sample period, accompanied by a noticeable widening of the distribution. In Colombia, labor-augmenting productivity also rises gradually with moderate dispersion. In Chile, the distribution is more volatile early in the sample but shows a clear upward shift in later years. The figures suggest that Hicks-neutral and labor-augmenting productivity are only weakly correlated over time, indicating that the two components capture distinct dimensions of technological change rather than moving closely together. Overall, these patterns suggest that technology dynamics are driven primarily by changes in labor-augmenting productivity rather than factor-neutral technological shifts.

Figure O.2 presents scatter plots correlating productivity estimates from our unrestricted model ("Main") against those obtained from the homothetically separable model ("HS") across all fifteen country-industry cells. Identical estimates from these two models would fall exactly on the 45-degree line; deviations from the line indicate that the HS restriction alters the productivity measurements. In each panel, horizontal and vertical axes share the range of the main model estimates.

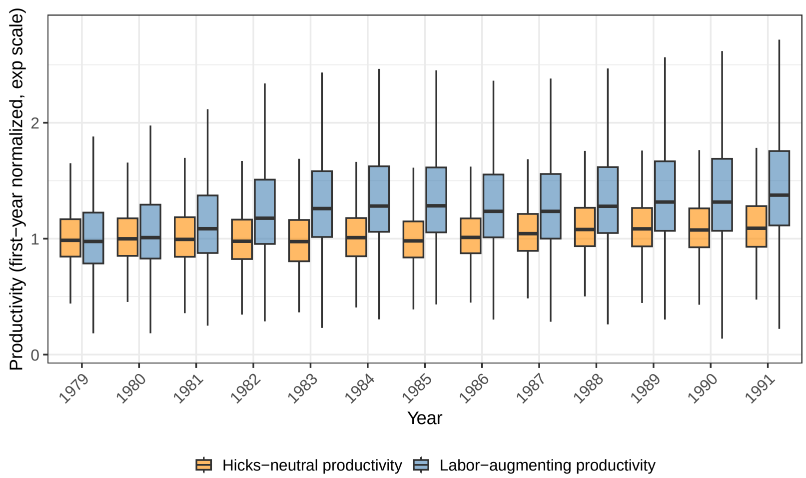
Table O.4: Median Estimates of Output Elasticities, RTS, and Elasticities of Substitution

	Main	HS	Main	HS	Main	HS	Main	HS	Main	HS
					CHINA					
	Food		*Textile*		*Chemic.*		*Mineral*		*Machin.*	
K	0.149	0.135	0.072	0.067	0.124	0.112	0.116	0.110	0.100	0.084
L	0.043	0.043	0.082	0.083	0.062	0.062	0.125	0.124	0.091	0.091
M	1.132	1.143	0.981	0.983	1.056	1.061	1.216	1.205	0.935	0.937
RTS	1.325	1.321	1.135	1.133	1.243	1.235	1.463	1.438	1.125	1.112
AES_{KL}	1.995	1.224	1.126	1.233	1.255	1.295	1.893	1.374	0.789	1.370
AES_{KM}	1.263	1.224	1.338	1.233	1.453	1.295	1.398	1.374	1.587	1.370
AES_{LM}	–0.530	–0.296	1.694	1.696	1.333	1.457	1.591	1.698	1.779	1.774
MES_{KL}	1.322	1.224	1.309	1.233	1.420	1.295	1.500	1.374	1.457	1.370
MES_{LK}	–0.228	–0.138	1.568	1.601	1.312	1.428	1.713	1.651	1.485	1.688
MES_{KM}	1.249	1.224	1.319	1.233	1.438	1.295	1.460	1.374	1.504	1.370
MES_{MK}	1.274	1.240	1.411	1.313	1.478	1.333	1.454	1.461	1.649	1.446
MES_{LM}	–0.249	–0.145	1.657	1.669	1.325	1.442	1.623	1.679	1.682	1.742
MES_{ML}	–0.346	–0.145	1.673	1.669	1.353	1.442	1.584	1.679	1.753	1.742
					COLOMBIA					
	Food		*Textile*		*Apparel*		*Wood*		*Metals*	
K	0.265	0.256	0.350	0.297	0.165	0.140	0.182	0.212	0.301	0.283
L	0.131	0.137	0.231	0.199	0.269	0.294	0.271	0.286	0.268	0.252
M	0.714	0.750	0.538	0.462	0.545	0.598	0.575	0.611	0.528	0.501
RTS	1.120	1.143	1.124	0.958	0.982	1.032	1.035	1.110	1.109	1.036
AES_{KL}	0.551	1.092	1.024	1.120	1.191	1.466	-0.029	1.083	1.727	1.168
AES_{KM}	1.566	1.092	1.870	1.120	2.166	1.466	1.992	1.083	1.476	1.168
AES_{LM}	–0.351	1.608	3.417	3.836	–0.284	4.202	–0.795	1.461	3.452	2.194
MES_{KL}	1.370	1.092	1.441	1.120	1.756	1.466	1.383	1.083	1.627	1.168
MES_{LK}	–0.159	1.378	1.926	2.112	1.549	2.740	–0.545	1.280	2.316	1.630
MES_{KM}	1.508	1.092	1.704	1.120	1.959	1.466	1.649	1.083	1.560	1.168
MES_{MK}	1.762	1.214	2.565	1.913	3.139	2.543	1.768	1.189	2.247	1.459
MES_{LM}	–0.181	1.497	2.860	2.971	–0.030	3.798	–0.703	1.389	3.028	1.908
MES_{ML}	0.048	1.497	3.159	2.971	0.370	3.798	–0.363	1.389	2.988	1.908
					CHILE					
	Food		*Textile*		*Apparel*		*Wood*		*Metals*	
K	0.148	0.107	0.265	0.124	0.319	0.286	0.192	0.208	0.224	0.019
L	0.152	0.154	0.158	0.144	0.276	0.170	0.161	0.182	0.198	0.218
M	0.789	0.801	0.612	0.555	1.115	0.688	0.637	0.720	0.611	0.594
RTS	1.095	1.063	1.044	0.823	1.733	1.144	0.989	1.109	1.075	0.831
AES_{KL}	1.408	1.334	1.357	1.023	2.928	1.255	2.645	1.328	2.606	1.069
AES_{KM}	1.831	1.334	1.694	1.023	2.024	1.255	1.470	1.328	2.693	1.069
AES_{LM}	2.091	1.658	2.093	2.847	–2.884	0.536	–2.740	1.546	–1.896	0.708
MES_{KL}	1.701	1.334	1.545	1.023	2.515	1.255	1.916	1.328	2.736	1.070
MES_{LK}	1.885	1.621	1.700	1.811	–0.577	0.833	-0.616	1.528	–0.032	1.023
MES_{KM}	1.767	1.334	1.621	1.023	2.150	1.255	1.601	1.328	2.711	1.073
MES_{MK}	1.928	1.410	1.812	1.696	0.937	1.140	0.760	1.375	1.696	1.051
MES_{LM}	2.002	1.629	1.902	2.573	–1.530	0.721	–1.565	1.498	–0.668	1.003
MES_{ML}	2.098	1.629	2.032	2.573	–1.733	0.721	–1.837	1.498	–0.737	1.003

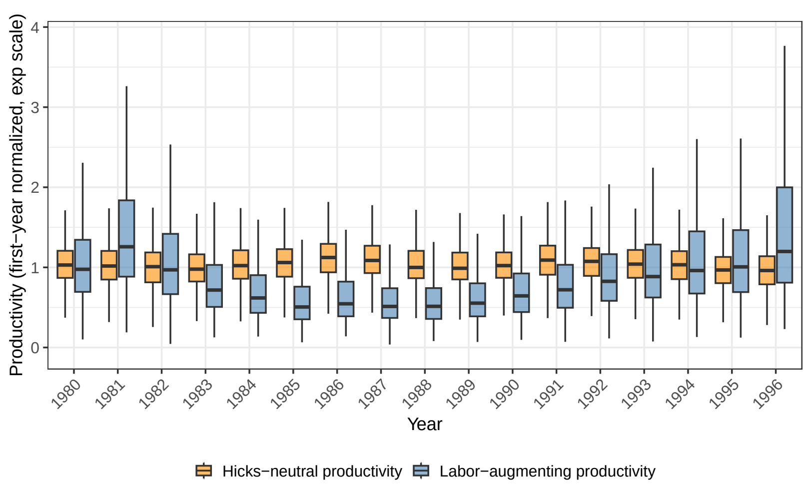
For China, both productivity measures align closely with the 45-degree line across all five industries, particularly in the textiles industry, indicating that the homothetic separability restriction has relatively little effect on the recovered productivity in the Chinese sample. The picture changes substantially for Colombia and Chile. In many industries, productivity estimates deviate noticeably from the 45-degree line, with the HS model systematically over- or under-predicting relative to the main model. In some cases (the textile, apparel, and metals industries of Colombia and the food industry of Chile), Hicks-neutral productivity remains reasonably close to the 45-degree line while labor-augmenting productivity may diverge markedly. The dispersion around the line is also considerably wider than in China, reflecting greater sensitivity of productivity recovery to the separability restriction. For some industries, such as the apparel industry of Chile, both productivity clouds are diffuse and displaced from the 45-degree line, indicating that the HS restriction substantially distorts productivity estimates. Most severe distortion is in the metals industry of Chile, where the majority of estimates from the HS model are very different and fall out of the figure range. Overall, the figure suggests that the homothetic separability restriction matters more for productivity measurement in Colombia and Chile than in China, and that its distortionary effect is more pronounced for labor-augmenting productivity than for Hicks-neutral productivity.



(a) China



(b) Colombia



(c) Chile

Figure O.1: Hicks-Neutral and Labor-Augmenting Productivity Variation and Trend

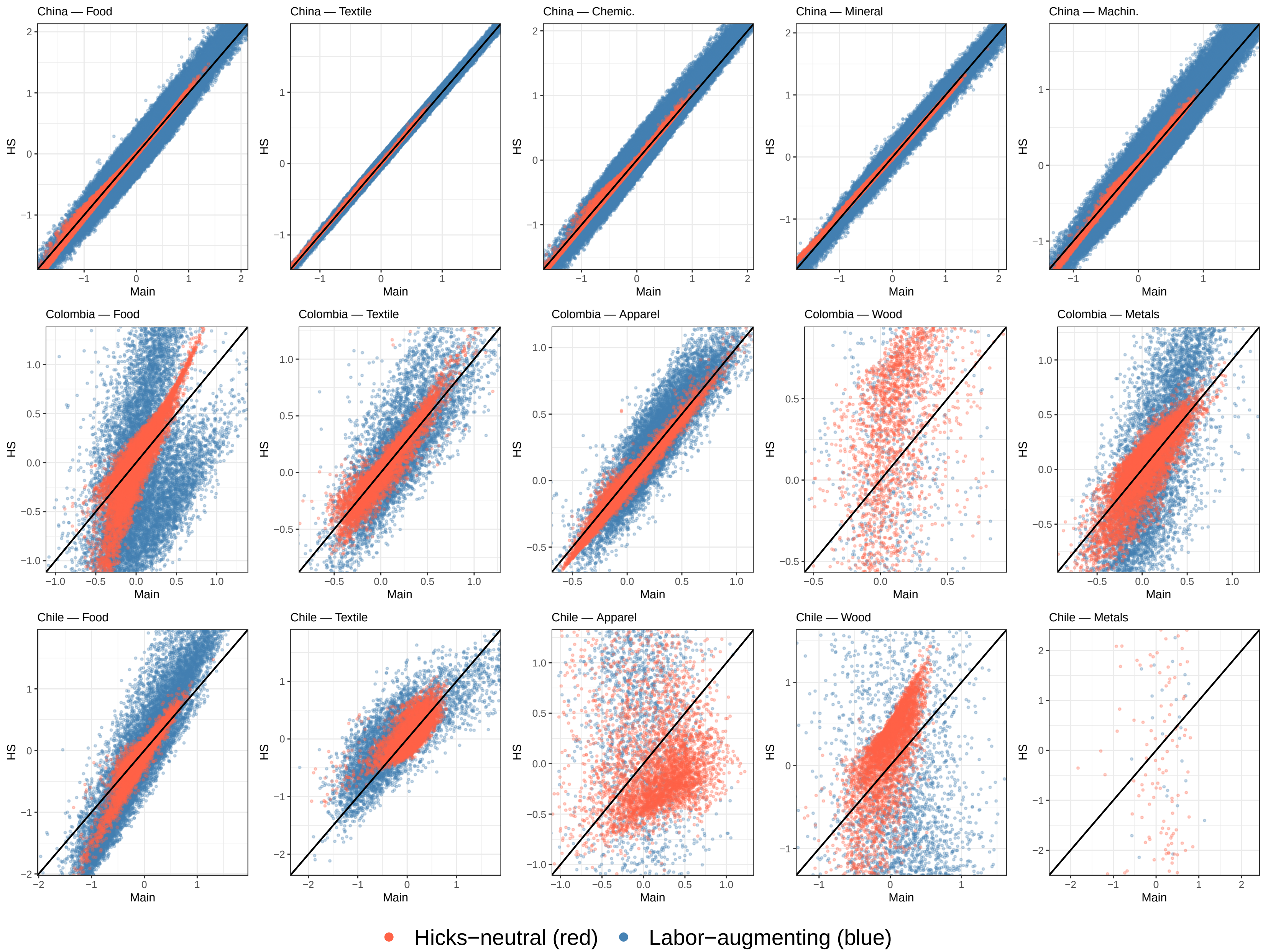


Figure O.2: Productivity Estimates from Not-Homothetically-Separable (Main) vs. Homothetically Separable (HS) Models