Supplementary Appendix to
Dynamic Responses to Oil Price Shocks: Conditional vs Unconditional (A)symmetry

Emir Malikov*

This Draft: September 30, 2015

Abstract

The impulse-response-function-based Wald test has been gaining wide popularity among researchers seeking to formally test for (a)symmetries in dynamic responses of various macroeconomic aggregates to oil price shocks. However, because the IRF-based Wald test is *conditional* on the magnitude of an oil price shock, it can sometimes prove to be impractical, especially when producing contrasting evidence for shocks of different sizes. To circumvent this problem, this paper suggests considering a nonparametric IRF-*density*-based test in addition to the Wald. The former allows the analysis of (a)symmetries in dynamic impulse responses to positive and negative oil price shocks of a wide range of magnitudes. The test permits inference about a general tendency of (a)symmetries in impulse responses as opposed to (a)symmetries pertinent to a shock of a given size only. The examined (a)symmetry is thus *unconditional* of the magnitude of a shock. Importantly, the testing procedure allows accounting for the relative likelihood of observing the disturbance of a given size.

Keywords: Asymmetry, Impulse Response, Job Creation and Destruction, Macroeconomy, Nonlinearity, Oil Price, SVAR

JEL Classification: C32, Q43

*Department of Economics, St. Lawrence University, Canton, NY; *Email:* emalikov@stlawu.edu.

Supplementary Appendix

A Estimation of Impulse Response Functions

This section describes the procedure used to compute unconditional impulse response functions (IRFs) of job creation and destruction to an oil price shock. Throughout this appendix, the "conditional IRF" and "unconditional IRF" refer to the IRF dependent on the pre-impulse history and the IRF independent of the pre-impulse history, respectively (for detail, also see Gallant et al., 1993; Koop et al., 1996).

1. Model (1) is estimated equation by equation via OLS with the parameter estimates and residuals for each equation saved in the memory. Denote these parameter estimates as $\left(\widehat{\mathcal{B}}_1, \widehat{\mathcal{B}}_2, \widehat{\mathcal{B}}_3\right)$ and the residuals as $(\widehat{\boldsymbol{\epsilon}}_1, \widehat{\boldsymbol{\epsilon}}_2, \widehat{\boldsymbol{\epsilon}}_3)$.

2. Given some history $(x_{t-1}, \ldots, x_{t-p}, y_{1,t-1}, \ldots, y_{1,t-p}, y_{2,t-1}, \ldots, y_{2,t-p}) \equiv (\mathbf{x}_t, \mathbf{y}_{1,t}, \mathbf{y}_{2,t}) \in \boldsymbol{\Xi}^t$, generate two paths for x_t:
$$x_t^1 = (1, \mathbf{x}_t, \mathbf{y}_{1,t}, \mathbf{y}_{2,t})\, \widehat{\mathcal{B}}_1 + \delta$$
$$x_t^2 = (1, \mathbf{x}_t, \mathbf{y}_{1,t}, \mathbf{y}_{2,t})\, \widehat{\mathcal{B}}_1 + \widehat{\epsilon}_{1,t} \ ,$$
where δ is an oil price shock set to equal to a desired magnitude, and $\widehat{\epsilon}_{1,t}$ is randomly drawn (with replacement) from the empirical distribution of $\widehat{\boldsymbol{\epsilon}}_1$.

3. Given the two generated paths for x_t and associated with them two paths for x_t^*, generate two paths for $y_{1,t}$ and $y_{2,t}$:
$$y_{1,t}^1 = \left(1, x_t^1, \mathbf{x}_t, \mathbf{y}_{1,t}, \mathbf{y}_{2,t}, x_t^{*1}, \mathbf{x}_t^*\right) \widehat{\mathcal{B}}_2 + \widehat{\epsilon}_{2,t}$$
$$y_{1,t}^2 = \left(1, x_t^2, \mathbf{x}_t, \mathbf{y}_{1,t}, \mathbf{y}_{2,t}, x_t^{*2}, \mathbf{x}_t^*\right) \widehat{\mathcal{B}}_2 + \widehat{\epsilon}_{2,t}$$
$$y_{2,t}^1 = \left(1, x_t^1, \mathbf{x}_t, \mathbf{y}_{1,t}, \mathbf{y}_{2,t}, x_t^{*1}, \mathbf{x}_t^*\right) \widehat{\mathcal{B}}_3 + \widehat{\epsilon}_{3,t}$$
$$y_{2,t}^2 = \left(1, x_t^2, \mathbf{x}_t, \mathbf{y}_{1,t}, \mathbf{y}_{2,t}, x_t^{*2}, \mathbf{x}_t^*\right) \widehat{\mathcal{B}}_3 + \widehat{\epsilon}_{3,t} \ ,$$
where $\widehat{\epsilon}_{2,t}$ and $\widehat{\epsilon}_{3,t}$ are randomly drawn (with replacement) from the *joint* (!) empirical distribution of $\widehat{\boldsymbol{\epsilon}}_2$ and $\widehat{\boldsymbol{\epsilon}}_3$. It is crucial to draw $\widehat{\epsilon}_{2,t}$ and $\widehat{\epsilon}_{3,t}$ from the joint (not marginal) distribution because, given the identification scheme used in model (1), the latter are not structural but partially reduced-form innovations. Also note that the same residuals $\widehat{\epsilon}_{2,t}$ and $\widehat{\epsilon}_{3,t}$ are used to generate $\left(y_{1,t}^1, y_{1,t}^2\right)$ and $\left(y_{2,t}^1, y_{2,t}^2\right)$, respectively.

4. Update the information set to be used in forecasting x_{t+1}^1 and x_{t+1}^2:
$\left(1, x_t^1, x_{t-1}, \ldots, x_{t-p+1}, y_{1,t}^1, y_{1,t-1}, \ldots, y_{1,t-p+1}, y_{2,t}^1, y_{2,t-1}, \ldots, y_{2,t-p+1}\right) \equiv \mathbf{Z}_{t+1}^1$ and
$\left(1, x_t^2, x_{t-1}, \ldots, x_{t-p+1}, y_{1,t}^2, y_{1,t-1}, \ldots, y_{1,t-p+1}, y_{2,t}^2, y_{2,t-1}, \ldots, y_{2,t-p+1}\right) \equiv \mathbf{Z}_{t+1}^2$.
Next, construct x_{t+1}^1 and x_{t+1}^2:
$$x_{t+1}^1 = \mathbf{Z}_{t+1}^1 \widehat{\mathcal{B}}_1 + \widehat{\epsilon}_{1,t+1}$$
$$x_{t+1}^2 = \mathbf{Z}_{t+1}^2 \widehat{\mathcal{B}}_1 + \widehat{\epsilon}_{1,t+1} \ ,$$
where $\widehat{\epsilon}_{1,t+1}$ is randomly drawn (with replacement) from the empirical distribution of $\widehat{\boldsymbol{\epsilon}}_1$. Again note that the same residual $\widehat{\epsilon}_{1,t+1}$ is used to generate both x_{t+1}^1 and x_{t+1}^2.

5. Repeat steps 3 and 4 $(H+1)$ times.

6. After N simulations of steps 2–5, estimate the *conditional* IRFs for job creation and destruction as follows:

$$I_{y_1}\left(h,\delta,\boldsymbol{\Xi}^t\right) = \frac{1}{N}\sum_{j=1}^{N}\left(y^1_{1,t+h,j} - y^2_{1,t+h,j}\right) \qquad \text{for } h = 0,\ldots,H$$

$$I_{y_2}\left(h,\delta,\boldsymbol{\Xi}^t\right) = \frac{1}{N}\sum_{j=1}^{N}\left(y^1_{2,t+h,j} - y^2_{2,t+h,j}\right) \qquad \text{for } h = 0,\ldots,H.$$

7. The *unconditional* IRFs are computed by repeating steps 2–6 for a set of random sub-samples (better yet, all) histories $\boldsymbol{\Xi}^t$ $(t = 1,\ldots,T)$ and then integrating over these histories:

$$I_{y_1}(h,\delta) = \frac{1}{T}\sum_{t=1}^{T} I_{y_1}\left(h,\delta,\boldsymbol{\Xi}^t\right) \qquad \text{for } h = 0,\ldots,H$$

$$I_{y_2}(h,\delta) = \frac{1}{T}\sum_{t=1}^{T} I_{y_1}\left(h,\delta,\boldsymbol{\Xi}^t\right) \qquad \text{for } h = 0,\ldots,H.$$

References

Gallant, A.R., Rossi, P., Tauchen, G. (1993). Nonlinear Dynamic Structure. *Econometrica* 61, 876–907.

Koop, G., Pesaran, M.H., Potter, S.M. (1996). Impulse Response Analysis in Nonlinear Multivariate Models. *Journal of Econometrics* 74, 119–147.

B Estimation of the IRF Covariance Matrix

This section describes the procedure used to estimate of the covariance matrix for the IRF estimators, i.e., $\mathbb{E}[(\widehat{\mathbf{I}}-\mathbf{I})(\widehat{\mathbf{I}}-\mathbf{I})']$, where $\widehat{\mathbf{I}} = \left[I_y(0,\delta),\ldots,I_y(H,\delta),I_y(0,-\delta),\ldots,I_y(H,-\delta)\right]'$. I use the recursive-design bootstrap.

1. Given the parameter estimates $\left(\widehat{\mathcal{B}}_1,\widehat{\mathcal{B}}_2,\widehat{\mathcal{B}}_3\right)$ and residuals $(\widehat{\boldsymbol{\epsilon}}_1,\widehat{\boldsymbol{\epsilon}}_2,\widehat{\boldsymbol{\epsilon}}_3)$ from model (1) as well as a randomly chosen history $\boldsymbol{\Xi}^b$, generate pseudo-series for x, y_1 and y_2 of length T. Denote the generated pseudo-series as $\left(\mathbf{x}^b,\mathbf{y}^b_1,\mathbf{y}^b_2\right)$.

2. Repeat steps 1–7 described in Appendix A using pseudo-series $\left(\mathbf{x}^b,\mathbf{y}^b_1,\mathbf{y}^b_2\right)$ in place of actual data series. Store the computed bootstrap estimates of unconditional IRFs in $\widehat{\mathbf{I}}^b = \left[I^b_y(0,\delta),\ldots,I^b_y(H,\delta),I^b_y(0,-\delta),\ldots,I^b_y(H,-\delta)\right]'$.

3. Repeat steps 1 and 2 B times.

4. Using the bootstrap sample of B estimates of $\widehat{\mathbf{I}}^b$, compute the IRF covariance matrix $\widehat{\boldsymbol{\Omega}}$.

C IRF-density-based Test

This section describes (i) the procedure for computing the Li et al.'s (2009) $\mathcal{T}_n$ test statistic, (ii) the procedure for obtaining the bootstrap distribution of the $\mathcal{T}_n$ test statistic, and (iii) the "inverse transform" procedure.

C.1 Computation of the $\mathcal{T}_n$ Test Statistic

To construct the test statistic given by

$$\mathcal{T}_n = \frac{N\left(\lambda_0 \cdots \lambda_H\right)^{1/2}}{\widehat{\sigma}_n} \widehat{\mathcal{I}}_n,$$

we first need to compute $\widehat{\mathcal{I}}_n$ and $\widehat{\sigma}_n$.

Following Li et al. (2009), for each H, the estimate of the integrated square difference between two densities is computed as follows:

$$\widehat{\mathcal{I}}_n = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j(\neq i)=1}^{N} \mathcal{K}(\boldsymbol{\lambda}, \mathbf{I}_i^+, \mathbf{I}_j^+) + \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j(\neq i)=1}^{N} \mathcal{K}(\boldsymbol{\lambda}, \mathbf{I}_i^-, \mathbf{I}_j^-) - \frac{1}{N^2}\left[\sum_{i=1}^{N}\sum_{j=1}^{N} \mathcal{K}(\boldsymbol{\lambda}, \mathbf{I}_i^+, \mathbf{I}_j^-) + \sum_{i=1}^{N}\sum_{j=1}^{N} \mathcal{K}(\boldsymbol{\lambda}, \mathbf{I}_j^+, \mathbf{I}_i^-)\right],$$

where $\boldsymbol{\lambda} = [\lambda_0, \ldots, \lambda_H]'$ is an $(H+1) \times 1$ vector of bandwidths, $\mathbf{I}_i^+ = \big[I_y(0, \delta_i), \ldots, I_y(H, \delta_i)\big]'$ and $\mathbf{I}_i^- = \big[-I_y(0, -\delta_i), \ldots, -I_y(H, -\delta_i)\big]'$ are the $(H+1) \times 1$ vectors of impulse responses to a positive and negative oil price shock of the ith magnitude at horizons $h = 0, \ldots, H$, respectively. There is a total of N oil price magnitudes. Given that in my design $\delta_i \in \left\{\frac{1}{3000}3\sigma, \frac{2}{3000}3\sigma, \ldots, \frac{2999}{3000}3\sigma, 3\sigma\right\}$, here $N = 3000$. The product kernel $\mathcal{K}(\cdot)$ is defined as $\mathcal{K}(\boldsymbol{\lambda}, \mathbf{I}_i^+, \mathbf{I}_j^+) = \prod_{s=0}^{H} \lambda_s^{-1} K\left(\frac{I_y(s,\delta_i) - (-I_y(s,-\delta_j))}{\lambda_s}\right)$.

The estimate of the variance of $\mathcal{I}_n$ is computed for each H as follows:

$$\widehat{\sigma}_n^2 = 2N^2 \lambda_0 \cdots \lambda_H \Big[\frac{1}{N^2(N-1)^2} \sum_{i=1}^{N} \sum_{j(\neq i)=1}^{N} \mathcal{K}^2(\boldsymbol{\lambda}, \mathbf{I}_i^+, \mathbf{I}_j^+) + \frac{1}{N^2(N-1)^2} \sum_{i=1}^{N} \sum_{j(\neq i)=1}^{N} \mathcal{K}^2(\boldsymbol{\lambda}, \mathbf{I}_i^-, \mathbf{I}_j^-) + \frac{1}{N^4}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathcal{K}^2(\boldsymbol{\lambda}, \mathbf{I}_i^+, \mathbf{I}_j^-) + \frac{1}{N^4}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathcal{K}^2(\boldsymbol{\lambda}, \mathbf{I}_j^+, \mathbf{I}_i^-)\Big].$$

For each H, the bandwidths $\boldsymbol{\lambda}$ are selected via the data-driven cross-validation. That is, we cross-validate the entire vector $\boldsymbol{\lambda}$ H times.

C.2 Bootstrap Procedure

Despite that $\mathcal{T}_n$ is asymptotically normal, the normal limiting distribution may be a poor approximation for the finite-sample distribution of the test statistic (Li et al., 2009). To mitigate this problem, for inference, I obtain the bootstrap distribution of $\mathcal{T}_n$.

Following the procedure proposed by Li et al. (2009):

1. Pool the two samples and let

$$\widehat{\mathbf{I}}_i = \begin{cases} \mathbf{I}_i^+ & \text{for } i = 1, \ldots, N \\ \mathbf{I}_i^- & \text{for } i = (N+1), \ldots, 2N. \end{cases}$$

2. From $\left\{\widehat{\mathbf{I}}_i\right\}_{i=1}^{2N}$, randomly draw (with replacement) N observations of $\widehat{\mathbf{I}}_i$ and store it as $\left\{\widehat{\mathbf{I}}_i^{+,\mathrm{boot}}\right\}_{i=1}^{N}$.

3. From $\left\{\widehat{\mathbf{I}}_i\right\}_{i=1}^{2N}$, randomly draw (with replacement) another N observations of $\widehat{\mathbf{I}}_i$ and store it as $\left\{\widehat{\mathbf{I}}_i^{-,\text{boot}}\right\}_{i=1}^{N}$.

4. Using bootstrap samples $\left\{\widehat{\mathbf{I}}_i^{+,\text{boot}}\right\}_{i=1}^{N}$ and $\left\{\widehat{\mathbf{I}}_i^{-,\text{boot}}\right\}_{i=1}^{N}$, compute the bootstrap test statistic $\mathcal{T}_n^{\text{boot}}$.

5. Repeat steps 2–4 B times.

6. Use the bootstrap sample of test statistics $\left\{\mathcal{T}_{n,b}^{\text{boot}}\right\}_{b=1}^{B}$ to approximate the null distribution of $\mathcal{T}_n$. Compute empirical p-values via $B^{-1}\sum_{b=1}^{B}\mathbb{I}\left\{\mathcal{T}_{n,b}^{\text{boot}} > \mathcal{T}_n\right\}$, where $\mathbb{I}\{\cdot\}$ is an indicator function.

C.3 Inverse Transform

This subsection describes the "inverse transform" procedure used to generate a pseudo-random sample of absolute magnitudes of oil price shocks ranging from $\frac{1}{3000}3\sigma$ to 3σ, which has the same distribution as the absolute values of historical oil price disturbances estimated by equation (1a).

First, note that the traditional inverse transform sampling is not feasible here, since the method is designed to generate a sample based on a known distribution. Since in our case the true distribution of absolute oil price disturbances is unknown, an empirical distribution is used instead.

1. Equation (1a) is estimated via OLS with the residuals $\widehat{\boldsymbol{\epsilon}}_1$ saved in the memory. These are the estimates of historical oil price shocks; abs$[\widehat{\boldsymbol{\epsilon}}_1]$ is then a vector of historical magnitudes.

2. Estimate cumulative distribution of abs$[\widehat{\boldsymbol{\epsilon}}_1]$ via the kernel method. The kernel used is the second-order Gaussian. The optimal bandwidth is selected via the least squares cross-validation.

3. Evaluate the estimated cumulative distribution at $\{e_i\}_{i=1}^{3000} = \frac{1}{3000}3\sigma, \ldots, \frac{2999}{3000}3\sigma, 3\sigma$ using the selected bandwidth from step 2. The predicted distribution is a monotonically increasing vector of corresponding cumulative probabilities $\{p_i\}_{i=1}^{3000} = p_1, \ldots, p_{2999}, p_{3000}$.

4. Use the cdf predicted in step 3 in a traditional inverse transformation sampling for a discrete random variable:

   (a) Generate a random number $U \sim \mathbb{U}(0,1)$

   (b) Transform U into E as follows: $E = e_i$ if $p_{i-1} \leq U < p_i$

   (c) Repeat steps (a) and (b) 3000 times

   The resulting vector $\mathbf{E} \equiv (E_1, \ldots, E_{3000})$ is a 3000×1 vector of absolute magnitudes of oil price innovations that has the very same distribution as abs$[\widehat{\boldsymbol{\epsilon}}_1]$.

5. Disturb model (1) with positive and negative oil price shocks of magnitudes $\delta \in \mathbf{E}$. This will produce two empirical distributions of impulse responses to positive and negative oil price disturbances for each job flow at any given horizon.

References

Li, Q., Maasoumi, E., Racine, J.S. (2009). A Nonparametric Test for Equality of Distributions with Mixed Categorical and Continuous Data. *Journal of Econometrics* 148, 186–200.

D Results for Mork's (1989) Oil Price Increase Measure

Mork's (1989) oil price increase is another popular nonlinear oil price transformation used in the literature. The measure is defined as:

$$x_t^+ = \max\{0, o_t - o_{t-1}\} = \max\{0, x_t\},$$

where o_t is the log of the real oil price. Here, the variable is censored to disregard negative oil price growth. Mork (1989) hypothesizes that what matters for the U.S. economy is only oil price increases: they are the ones that contribute to recessions, while oil price decreases bear no stimulating momentum for an economy whatsoever.

I repeat the analysis described in Sections 3 and 4 of the paper using Mork's (1989) transformation [i.e., setting $x_t^* = x_t^+$ in model (1)]. The IRF-based Wald test of conditional (a)symmetry in impulse responses consistently rejects the null of symmetry (at the 5% significance level) for all three manufacturing job flows at all horizons $h = 0, \ldots, 8$, regardless of the magnitude of an oil price shock (one or two st.d.). The corresponding p-values are all of the order 10^{-4} or less and are, therefore, unreported. Testing for unconditional (a)symmetry using the IRF-density-based test provides further support to the evidence that manufacturing job flows respond to oil price shocks asymmetrically. Table D.1 reports the estimates of the corresponding test statistic. The test statistics are all large in magnitude with the corresponding bootstrap p-values being consistently less than 10^{-6}, leading to an overwhelming rejection of the respective null hypotheses of symmetry in impulse responses as far as at a two-year horizon. The latter reinforces the main finding of the paper, whereby dynamic responses of job flows to oil shocks are found to be generally asymmetric.

Table D.1. IRF-Density-based Tests of (Unconditional) Asymmetry for $x_t^* = x_t^+$

	Uniform Distr. of Shocks			*Emp. Distr. of Shocks*		
Horizon	JC	JD	NG	JC	JD	NG
0	116.57	172.25	173.18	82.70	71.43	68.57
1	213.95	139.73	139.64	75.02	80.86	69.92
2	138.02	113.54	117.36	82.37	80.35	68.02
3	77.05	75.10	81.52	60.92	60.82	60.63
4	56.53	56.43	64.70	59.80	79.88	77.64
5	48.06	45.96	49.49	67.85	83.54	59.67
6	38.79	36.68	39.10	59.14	62.22	73.28
7	31.89	29.57	32.19	71.69	60.87	67.17
8	27.48	25.05	28.18	59.29	76.82	65.80

The reported are $\mathcal{T}_n$ test statistics.

References

Mork, K.A. (1989). Oil and the Macroeconomy When Price Go Up and Down: An Extension of Hamilton's Results. *Journal of Political Economy* 97(3), 740–744.